Exercise Collection

Decision Theory – Decision Making under Uncertainty

English

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Table of Contents

1 Level 1: Easy **2**

Exercise 1.1 2

Exercise 1.2 3

2 Level 2: Medium **5**

Exercise 2.1 5

Exercise 2.2 6

3 Level 3: Hard **8**

Exercise 3.1 8

Exercise 3.2 9

1 Level 1: Easy

Exercise 1.1 – Maximax and Maximin

Exercise 1.1

An investor is evaluating three investment options. The expected returns (in thousand euros) depend on economic development:

Alternative	Boom	Stagnation	Recession
Stocks	80	20	−30
Bonds	30	25	15
Money market	15	15	15

(a) Apply the **Maximax** rule and justify the decision.

(b) Apply the **Maximin** rule and justify the decision.

(c) Explain the risk attitude behind each rule.

(d) Which alternative would a highly risk-averse investor choose?

Solution: Exercise 1.1

(a) Maximax – best possible outcome per alternative:

Alternative	Boom	Stagnation	Recession	**Maximum**
Stocks	80	20	−30	**80**
Bonds	30	25	15	30
Money market	15	15	15	15

⇒ **Stocks** (maximum = 80). An optimist betting on a boom chooses the option with the highest potential return.

(b) Maximin – best minimum (worst case):

Alternative	Boom	Stagnation	Recession	**Minimum**
Stocks	80	20	−30	−30
Bonds	30	25	15	15
Money market	15	15	15	**15**

max(min) = 15 for money market *and* bonds. ⇒ **money market or bonds**. Since both have the same minimum, bonds is preferable (higher maximum).

(c) Risk attitude:

- Maximax: *optimistic* – always expects the best state of nature.

- Maximin: *pessimistic* – always expects the worst state of nature (Wald criterion).

(d) Risk-averse investor: **Money market** – a guaranteed 15 thousand euros with no risk of loss.

Exercise 1.2 – Laplace and Hurwicz

Exercise 1.2

A company plans to launch a new product. The forecast contribution margins (thousand euros) depending on market demand:

Alternative	High demand	Medium demand	Low demand	No demand
Mass production	200	120	30	−50
Batch production	140	100	60	0
Small batch	80	70	65	50

(a) Apply the **Laplace** rule.

(b) Apply the **Hurwicz** rule for $\alpha = 0.4$ and $\alpha = 0.7$.

(c) For which value of α is small-batch production exactly as attractive as mass production?

Solution: Exercise 1.2

(a) Laplace ($p_j = 1/4 = 0.25$):

$$\bar{e}_{\text{Mass}} = (200 + 120 + 30 - 50)/4 = 300/4 = 75.0$$
$$\bar{e}_{\text{Batch}} = (140 + 100 + 60 + 0)/4 = 300/4 = 75.0$$
$$\bar{e}_{\text{Small}} = (80 + 70 + 65 + 50)/4 = 265/4 = 66.25$$

Tie: **Mass production = Batch production** (both 75.0).
(b) Hurwicz $H_i = \alpha \cdot \max + (1-\alpha) \cdot \min$:
For $\alpha = 0.4$:

$$H_{\text{Mass}} = 0.4 \cdot 200 + 0.6 \cdot (-50) = 80 - 30 = 50$$
$$H_{\text{Batch}} = 0.4 \cdot 140 + 0.6 \cdot 0 = 56$$
$$H_{\text{Small}} = 0.4 \cdot 80 + 0.6 \cdot 50 = 32 + 30 = 62$$

$\Rightarrow$ **Small batch** ($H = 62$).
For $\alpha = 0.7$:

$$H_{\text{Mass}} = 0.7 \cdot 200 + 0.3 \cdot (-50) = 140 - 15 = 125$$
$$H_{\text{Batch}} = 0.7 \cdot 140 + 0.3 \cdot 0 = 98$$
$$H_{\text{Small}} = 0.7 \cdot 80 + 0.3 \cdot 50 = 56 + 15 = 71$$

$\Rightarrow$ **Mass production** ($H = 125$).
(c) Threshold α^*: Small batch = Mass production:

$$\alpha \cdot 80 + (1-\alpha) \cdot 50 = \alpha \cdot 200 + (1-\alpha) \cdot (-50)$$

$80\alpha + 50 - 50\alpha = 200\alpha - 50 + 50\alpha \quad \Rightarrow \quad 30\alpha + 50 = 250\alpha - 50 \quad \Rightarrow \quad 220\alpha = 100 \quad \Rightarrow \quad \alpha^* = \frac{100}{220}$

For $\alpha < 0.455$: small batch. For $\alpha > 0.455$: mass production.

2 Level 2: Medium

Exercise 2.1 – Savage-Niehans and Method Comparison

Exercise 2.1

A logistics company must decide on a transport route. Costs (thousand euros) depending on traffic volume (minimization problem):

Alternative	Low	Normal	High	Very high
Route A (highway)	40	60	90	140
Route B (country road)	50	55	70	80
Route C (rail)	65	65	65	65

(a) Construct the regret matrix (Savage-Niehans).

(b) Which route does the Savage-Niehans rule recommend?

(c) Also apply Maximin and Laplace (note: minimization!).

(d) Create a summary table of all three methods and give a recommendation.

Solution: Exercise 2.1

(a) Regret matrix:
For minimization: $r_{ij} = e_{ij} - \min_k e_{kj}$ (column minimum instead of maximum).
Column minima: Low=40, Normal=55, High=65, Very high=65.

Alternative	Low	Normal	High	Very high	**max regret**
Route A	$40-40=0$	$60-55=5$	$90-65=25$	$140-65=75$	**75**
Route B	$50-40=10$	$55-55=0$	$70-65=5$	$80-65=15$	15
Route C	$65-40=25$	$65-55=10$	$65-65=0$	$65-65=0$	**25**

(b) Savage-Niehans: $\min(\text{max regret}) = 15 \Rightarrow$ **Route B**.
(c) Maximin (= Minimax for minimization):
Row maximum: A=140, B=80, C=65. $\min(\max) = 65 \Rightarrow$ **Route C**.
Laplace ($\bar{e}_i = \frac{1}{4}\sum_j e_{ij}$): $\bar{e}_A = (40+60+90+140)/4 = 82.5$, $\bar{e}_B = (50+55+70+80)/4 = 63.75$, $\bar{e}_C = 65$. $\min(\bar{e}) = 63.75 \Rightarrow$ **Route B**.
(d) Summary:

Method	Recommendation	Value
Maximin/Minimax	Route C	max. cost = 65
Laplace	Route B	avg = 63.75
Savage-Niehans	Route B	max. regret = 15

Recommendation: Route B – recommended by two methods and offers a good balance between cost and risk.

Exercise 2.2 – Expected Value and Sensitivity

Exercise 2.2

A pharmaceutical company is evaluating three research projects. Expected profits (million euros):

Project	Success	Partial success	Failure
Project X	500	100	−200
Project Y	300	150	50
Project Z	200	180	120

The research department estimates: $P(\text{Success}) = 0.3$, $P(\text{Partial success}) = 0.5$, $P(\text{Failure}) = 0.2$.

(a) Calculate the expected value for all three projects.

(b) Which project does the expected-value rule recommend?

(c) By how many percentage points would $P(\text{Success})$ need to fall for Project Y to become more attractive than Project X? (Assumption: $P(\text{Partial success})$ remains constant, Failure receives the remaining probability.)

(d) Additionally apply Maximin and compare.

Solution: Exercise 2.2

(a) Expected values:

$$E[X] = 0.3 \cdot 500 + 0.5 \cdot 100 + 0.2 \cdot (-200) = 150 + 50 - 40 = \mathbf{160}$$
$$E[Y] = 0.3 \cdot 300 + 0.5 \cdot 150 + 0.2 \cdot 50 = 90 + 75 + 10 = \mathbf{175}$$
$$E[Z] = 0.3 \cdot 200 + 0.5 \cdot 180 + 0.2 \cdot 120 = 60 + 90 + 24 = \mathbf{174}$$

(b) $\Rightarrow$ **Project Y** ($E = 175$ million euros).

(c) Sensitivity analysis:

Let p be the new success probability (Failure $= 0.2 + (0.3 - p)$):

$$E[X](p) = p \cdot 500 + 0.5 \cdot 100 + (0.5 - p) \cdot (-200) = 500p + 50 - 100 + 200p = 700p - 50$$
$$E[Y](p) = p \cdot 300 + 0.5 \cdot 150 + (0.5 - p) \cdot 50 = 300p + 75 + 25 - 50p = 250p + 100$$

Equation $E[X] = E[Y]$:

$$700p - 50 = 250p + 100 \quad \Rightarrow \quad 450p = 150 \quad \Rightarrow \quad p^* = \frac{1}{3} \approx 0.333$$

P(Success) must fall from 0.30 to below 0.333 – so already a **minimal decrease** makes Y more attractive. At $p = 0.3$, $E[Y] = 175 > E[X] = 160$ (Y is already better).
Check: Y is already better. Ask the reverse question: above which p does X become better than Y? $p > 1/3 \approx 0.333$ – so if p were to rise from 0.3 to above 0.333.
(d) Maximin: Minima: X=−200, Y=50, Z=120. ⇒ **Project Z** (safest minimum).
Comparison: expected value → Y; Maximin → Z. A risk-seeking decision maker chooses Y, a risk-averse one Z.

3 Level 3: Hard

Exercise 3.1 – All Six Methods Compared

Exercise 3.1

An energy company is deciding on the expansion of its generation capacity. Profit (million euros) depending on the development of energy demand:

Alternative	Strongly rising	Moderate	Stagnant	Declining
Large-scale solar	300	180	80	−40
Offshore wind	250	200	90	0
Gas backup	160	140	120	100
Nuclear	220	220	220	220

Expert probabilities: $P = (0.25;\ 0.35;\ 0.25;\ 0.15)$.

(a) Apply all six methods (Maximax, Maximin, Laplace, Hurwicz with $\alpha = 0.6$, Savage-Niehans, expected value).

(b) Create a complete comparison table.

(c) Which alternative receives the most votes?

(d) Interpret the result: when would one nevertheless prefer a method with fewer votes?

Solution: Exercise 3.1

(a) All methods:

Maximax: max: Solar=300, Wind=250, Gas=160, Nuclear=220. $\Rightarrow$ **Solar** (300).
Maximin: min: Solar=−40, Wind=0, Gas=100, Nuclear=220. $\Rightarrow$ **Nuclear** (220).
Laplace: $\bar{e} = \frac{1}{4}\sum_j$: Solar=$(300+180+80-40)/4 = 130$, Wind=$(250+200+90+0)/4 = 135$, Gas=$(160+140+120+100)/4 = 130$, Nuclear=220. $\Rightarrow$ **Nuclear** (220).
Hurwicz ($\alpha = 0.6$)**:** $H_i = 0.6 \cdot \max + 0.4 \cdot \min$: $H_{\text{Solar}} = 0.6 \cdot 300 + 0.4 \cdot (-40) = 180 - 16 = 164$, $H_{\text{Wind}} = 0.6 \cdot 250 + 0.4 \cdot 0 = 150$, $H_{\text{Gas}} = 0.6 \cdot 160 + 0.4 \cdot 100 = 96 + 40 = 136$, $H_{\text{Nuclear}} = 0.6 \cdot 220 + 0.4 \cdot 220 = 220$. $\Rightarrow$ **Nuclear** (220).
Savage-Niehans: Column optima: 300, 220, 220, 220.

	Strong	Moderate	Stagn.	Declining	max regret
Solar	0	40	140	260	**260**
Wind	50	20	130	220	**220**
Gas	140	80	100	120	**140**
Nuclear	80	0	0	0	80

$\Rightarrow$ **Nuclear** (max. regret = 80).
Expected value: $E[\text{Solar}] = 0.25 \cdot 300 + 0.35 \cdot 180 + 0.25 \cdot 80 + 0.15 \cdot (-40) = 75 + 63 + 20 - 6 = 152$
$E[\text{Wind}] = 0.25 \cdot 250 + 0.35 \cdot 200 + 0.25 \cdot 90 + 0.15 \cdot 0 = 62.5 + 70 + 22.5 + 0 = 155$
$E[\text{Gas}] = 0.25 \cdot 160 + 0.35 \cdot 140 + 0.25 \cdot 120 + 0.15 \cdot 100 = 40 + 49 + 30 + 15 = 134$
$E[\text{Nuclear}] = 220$ (constant). $\Rightarrow$ **Nuclear** (220).
(b) Comparison table:

Method	Recommendation	Value	Votes
Maximax	Solar	300	1
Maximin	Nuclear	220	
Laplace	Nuclear	220	
Hurwicz ($\alpha = 0.6$)	Nuclear	220	
Savage-Niehans	Nuclear	80	
Expected value	Nuclear	220	5

(c) Majority vote: **Nuclear** wins with 5 out of 6 votes.
(d) Interpretation: Despite the clear majority for nuclear, one might choose solar if one is *very optimistic* and prioritizes the high upside potential (300) – e.g., for a company that already has existential debt levels and is pursuing an "all-or-nothing" strategy (Maximax). Furthermore: at a low α in Hurwicz or very high success probabilities, wind or solar could become more attractive.

Exercise 3.2 – Extended Problem: Minimization and Sensitivity

Exercise 3.2

A production manager chooses between four manufacturing strategies. Costs (thousand euros) depending on demand development (minimization problem):

Strategy	Boom	Normal	Weak	Crisis
Full automation	80	90	140	200
Partial automation	100	100	110	130
Flexible manual	120	115	105	90
Outsourcing	110	105	100	95

Probabilities: $P = (0.30;\ 0.40;\ 0.20;\ 0.10)$.

(a) Apply Minimax (Maximin for costs), Laplace, Savage-Niehans and the expected value.

(b) Construct the complete regret matrix.

(c) For which Hurwicz α value (minimization: $H_i = \alpha \cdot \min_j + (1-\alpha) \cdot \max_j$) does the decision switch from outsourcing to full automation?

(d) Which strategy is most robust with respect to uncertainty in the probabilities? Justify.

Solution: Exercise 3.2

(a) Methods:
Minimax (min of the row maximum): max: Full=200, Partial=130, Manual=120, Out=110. $\min(200, 130, 120, 110) = 110 \Rightarrow$ **Outsourcing**.
Laplace ($\bar{c}_i = \frac{1}{4}\sum_j c_{ij}$): Full=$(80+90+140+200)/4 = 127.5$, Partial=$(100+100+110+130)/4 = 110$, Manual=$(120+115+105+90)/4 = 107.5$, Out=$(110+105+100+95)/4 = 102.5$. min $= 102.5 \Rightarrow$ **Outsourcing**.
Expected value: $E[\text{Full}] = 0.3 \cdot 80 + 0.4 \cdot 90 + 0.2 \cdot 140 + 0.1 \cdot 200 = 24 + 36 + 28 + 20 = 108$
$E[\text{Partial}] = 0.3 \cdot 100 + 0.4 \cdot 100 + 0.2 \cdot 110 + 0.1 \cdot 130 = 30 + 40 + 22 + 13 = 105$
$E[\text{Manual}] = 0.3 \cdot 120 + 0.4 \cdot 115 + 0.2 \cdot 105 + 0.1 \cdot 90 = 36 + 46 + 21 + 9 = 112$
$E[\text{Out}] = 0.3 \cdot 110 + 0.4 \cdot 105 + 0.2 \cdot 100 + 0.1 \cdot 95 = 33 + 42 + 20 + 9.5 = 104.5$
min $= 104.5 \Rightarrow$ **Outsourcing**.
(b) Regret matrix ($r_{ij} = c_{ij} - \min_k c_{kj}$):
Column minima: Boom=80, Normal=90, Weak=100, Crisis=90.

	Boom	Normal	Weak	Crisis	max regret
Full autom.	0	0	40	110	**110**
Partial autom.	20	10	10	40	40
Flex. manual	40	25	5	0	40
Outsourcing	30	15	0	5	30

Savage-Niehans: $\min(110, 40, 40, 30) = 30 \Rightarrow$ **Outsourcing**.
(c) Hurwicz threshold ($H_i = \alpha \cdot \min_j c_{ij} + (1-\alpha) \cdot \max_j c_{ij}$):
min: Full=80, Out=95. max: Full=200, Out=110.

$$H_{\text{Full}} = 80\alpha + 200(1-\alpha) = 200 - 120\alpha$$
$$H_{\text{Out}} = 95\alpha + 110(1-\alpha) = 110 - 15\alpha$$

Tie: $200 - 120\alpha = 110 - 15\alpha \Rightarrow 90 = 105\alpha \Rightarrow \alpha^* = \frac{90}{105} \approx$ **0.857**
For $\alpha > 0.857$ (very optimistic about low costs), full automation wins.
(d) Most robust strategy: **Outsourcing** – it has the smallest maximum regret (30), the lowest Laplace average (102.5) and the best expected value (104.5). Under uncertainty about the probabilities, outsourcing remains stable near the

front across all scenarios, without performing extremely poorly in any single state.