Decision Theory

Decision Under Uncertainty

Maximax, Maximin, Laplace, Hurwicz, Savage-Niehans, Expected value

English

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This document introduces decision theory and explains all six decision rules with a fully worked example.

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1 Introduction and Motivation

In der Praxis muss oft eine Entscheidung getroffen werden, ohne den zukünftigen **State of nature** zu kennen. Je nachdem, ob Probabilityen für die Zustände bekannt sind oder nicht, unterscheidet man:

- **Entscheidung unter Risk:** Probabilityen der States of nature bekannt $\Rightarrow$ Expected value-Methode.
- **Entscheidung unter Uncertainty:** keine Probabilityen bekannt $\Rightarrow$ Maximax, Maximin, Laplace, Hurwicz, Savage-Niehans.

Typische Anwendungen

- Produktionsentscheidung bei unbekannter Nachfrage
- Investition bei ungewisser Marktentwicklung
- Standortwahl bei unsicherer Infrastrukturentwicklung
- Lagerhaltungsentscheidungen bei schwankender Nachfrage

2 Key Concepts

Definition: Entscheidungsmatrix

A **decision matrix** $E = (e_{ij})$ with $i = 1, \ldots, m$ alternatives and $j = 1, \ldots, n$ states of nature contains the **outcome values** e_{ij} (profit, cost, utility) that arise when alternative i is chosen under state j.

	Z_1	Z_2	$\cdots$	Z_n
A_1	e_{11}	e_{12}	$\cdots$	e_{1n}
A_2	e_{21}	e_{22}	$\cdots$	e_{2n}
$\vdots$	$\vdots$	$\vdots$		$\vdots$
A_m	e_{m1}	e_{m2}	$\cdots$	e_{mn}

3 Decision Rules

3.1 Maximax Rule (Optimist)

Maximax Rule

Choose the alternative with the **best possible outcome**:

$$A^* = \arg\max_i \max_j e_{ij}$$

Stance: Extreme optimism – always expecting the most favourable state.

Critique: Completely ignores all unfavourable scenarios.

3.2 Maximin Rule (Wald Criterion, Pessimist)

Maximin Rule

Choose the alternative that performs best in the **worst case**:

$$A^* = \arg\max_i \min_j e_{ij}$$

Stance: Extreme pessimism – always expecting the least favourable state.
Critique: Very conservative; ignores positive scenarios.

3.3 Laplace Rule (Principle of Insufficient Reason)

Laplace Rule

Since no state is more likely than another, **equal probabilities** $p_j = 1/n$ are assumed. Choose the alternative with the highest average:

$$A^* = \arg\max_i \frac{1}{n} \sum_{j=1}^{n} e_{ij}$$

Stance: Neutral – all states treated equally.

3.4 Hurwicz Rule (Optimism Index)

Hurwicz Rule

Let $\alpha \in [0, 1]$ be the **optimism index**. Choose the alternative with the highest weighted value of best and worst outcome:

$$H_i = \alpha \cdot \max_j e_{ij} + (1 - \alpha) \cdot \min_j e_{ij}, \qquad A^* = \arg\max_i H_i$$

For $\alpha = 1$: Maximax. For $\alpha = 0$: Maximin.
Stance: Compromise between optimism and pessimism.

3.5 Savage-Niehans Rule (Minimax Regret)

Savage-Niehans Rule

Step 1 – Regret matrix: For each state j let $e_j^* = \max_i e_{ij}$ be the best achievable outcome. The **regret** is:

$$r_{ij} = e_j^* - e_{ij} \geq 0$$

Step 2 – Minimax Regret: Choose the alternative with the smallest maximum

regret:

$$A^* = \arg\min_i \max_j r_{ij}$$

Stance: Minimising ex-post regret ("What could I have done better?").

3.6 Expected Value Rule (Bayes Criterion)

Expected Value Rule

If probabilities $p_j \geq 0$ with $\sum_j p_j = 1$ are known:

$$E[A_i] = \sum_{j=1}^{n} p_j \cdot e_{ij}, \qquad A^* = \arg\max_i E[A_i]$$

Stance: Rational decision under risk – optimal choice in the long run.

4 Complete Example: Production Decision

4.1 Problem Statement

Example: Product Choice under Market Uncertainty

A company plans to launch a new product. Three alternatives are available. Profit (in thousand euros) depends on market conditions:

Alternative	Boom	Normal	Recession	Crisis
Product A	120	80	40	−10
Product B	90	70	60	20
Product C	60	55	50	40

Probabilities (for the expected value rule): $P(\text{Boom}) = 0.2$, $P(\text{Normal}) = 0.4$, $P(\text{Rec.}) = 0.3$, $P(\text{Crisis}) = 0.1$.

4.2 Method 1: Maximax

Row maximum per alternative:

$$\max_j e_{Aj} = 120, \quad \max_j e_{Bj} = 90, \quad \max_j e_{Cj} = 60$$

$$\Rightarrow \quad A^* = \textbf{Product A} \quad \text{(highest maximum: 120)}$$

4.3 Method 2: Maximin

Row minimum per alternative:

$$\min_j e_{Aj} = -10, \quad \min_j e_{Bj} = 20, \quad \min_j e_{Cj} = 40$$

$$\Rightarrow \quad A^* = \textbf{Product C} \quad \text{(best minimum: 40)}$$

4.4 Method 3: Laplace

Equal distribution $p_j = 1/4 = 0.25$:

$$\bar{e}_A = (120 + 80 + 40 + (-10))/4 = 230/4 = 57{,}5$$
$$\bar{e}_B = (90 + 70 + 60 + 20)/4 = 240/4 = 60{,}0$$
$$\bar{e}_C = (60 + 55 + 50 + 40)/4 = 205/4 = 51{,}25$$

$$\Rightarrow \quad A^* = \textbf{Product B} \quad \text{(highest average: 60.0)}$$

4.5 Method 4: Hurwicz ($\alpha = 0.5$)

$$H_i = 0{,}5 \cdot \max_j e_{ij} + 0{,}5 \cdot \min_j e_{ij}$$

$$H_A = 0{,}5 \cdot 120 + 0{,}5 \cdot (-10) = 60 - 5 = 55{,}0$$
$$H_B = 0{,}5 \cdot 90 + 0{,}5 \cdot 20 = 45 + 10 = 55{,}0$$
$$H_C = 0{,}5 \cdot 60 + 0{,}5 \cdot 40 = 30 + 20 = 50{,}0$$

$$\Rightarrow \quad A^* = \textbf{Product A or B} \quad \text{(tie: 55.0)}$$

For $\alpha = 0.3$ (more pessimistic): $H_A = 0{,}3 \cdot 120 + 0{,}7 \cdot (-10) = 36 - 7 = 29$, $H_B = 0{,}3 \cdot 90 + 0{,}7 \cdot 20 = 27 + 14 = 41$, $H_C = 0{,}3 \cdot 60 + 0{,}7 \cdot 40 = 18 + 28 = 46 \Rightarrow$ Product C wins.

4.6 Method 5: Savage-Niehans

Step 1 – Column optima: $e^*_{\text{Boom}} = 120$, $e^*_{\text{Normal}} = 80$, $e^*_{\text{Rez.}} = 60$, $e^*_{\text{Krise}} = 40$.

Step 2 – Regret matrix ($r_{ij} = e^*_j - e_{ij}$):

Alternative	Boom	Normal	Recession	Crisis	max regret
Product A	$120 - 120 = 0$	$80 - 80 = 0$	$60 - 40 = 20$	$40 - (-10) = 50$	**50**
Product B	$120 - 90 = 30$	$80 - 70 = 10$	$60 - 60 = 0$	$40 - 20 = 20$	30
Product C	$120 - 60 = 60$	$80 - 55 = 25$	$60 - 50 = 10$	$40 - 40 = 0$	**60**

$$\Rightarrow \quad A^* = \textbf{Product B} \quad \text{(smallest max. regret: 30)}$$

4.7 Method 6: Expected Value

With $p = (0.2;\ 0.4;\ 0.3;\ 0.1)$:

$$E[A] = 0{,}2 \cdot 120 + 0{,}4 \cdot 80 + 0{,}3 \cdot 40 + 0{,}1 \cdot (-10) = 24 + 32 + 12 - 1 = \mathbf{67{,}0}$$
$$E[B] = 0{,}2 \cdot 90 + 0{,}4 \cdot 70 + 0{,}3 \cdot 60 + 0{,}1 \cdot 20 = 18 + 28 + 18 + 2 = \mathbf{66{,}0}$$
$$E[C] = 0{,}2 \cdot 60 + 0{,}4 \cdot 55 + 0{,}3 \cdot 50 + 0{,}1 \cdot 40 = 12 + 22 + 15 + 4 = \mathbf{53{,}0}$$

$$\Rightarrow \quad A^* = \textbf{Product A} \quad (E[A] = 67.0)$$

4.8 Summary of Results

Method	Stance	Recommendation	Key value
Maximax	Optimistic	**Product A**	$\max = 120$
Maximin	Pessimistic	**Product C**	$\min = 40$
Laplace	Neutral	**Product B**	$\varnothing = 60.0$
Hurwicz ($\alpha = 0.5$)	Balanced	**A or B**	$H = 55.0$
Savage-Niehans	Regret	**Product B**	$r_{\max} = 30$
Expected value	Rational	**Product A**	$E = 67.0$
Majority vote (2 of 6)		**A, B**	

Interpretation: Product A is the best choice for risk-seeking decision makers (Maximax, EV). Product B suits risk-aware decision makers (Laplace, Savage). Product C is the safest choice (Maximin). The method should match the decision maker's risk attitude – there is no universally "correct" answer.

5 Tool Description

Decision Table Tool – All 6 Methods

The tool interactively computes all six decision rules for arbitrary matrices.

5.1 Input

- **Matrix size:** 2–8 alternatives, 2–8 states of nature.
- **Objective:** Maximisation (profit/utility) or minimisation (cost/loss).
- **Names:** Edit alternative and state names directly in the table.
- **Values:** Enter outcome values directly into the matrix.
- **Probabilities:** Optionally enable for the expected value method.
- **Hurwicz-α:** Slider from 0 (pessimistic) to 1 (optimistic).

5.2 Data Entry

1. **New (empty):** Empty matrix in the chosen size.
2. **Manual:** Enter values directly into the cells.
3. **CSV import:** File format:
   `;Z1-Name;Z2-Name;...`
   `P;0.2;0.4;...` (optional, for probabilities)

```
Alternative 1;Value11;Value12;...
Alternative 2;Value21;Value22;...
```

4. **CSV export:** Save the current matrix.

5.3 Output

- **Summary cards:** Each methodś recommendation with key value.
- **Vote count:** How many methods recommend each alternative?
- **Detail tables:**
  - Original decision matrix
  - Row extrema (Maximax/Maximin)
  - Laplace averages and expected values
  - Hurwicz calculation ($H = \alpha \cdot \text{best} + (1 - \alpha) \cdot \text{worst}$)
  - Full regret matrix (Savage-Niehans)

6 Summary

Key Points: Decision Under Uncertainty

1. The **decision matrix** contains outcome values for all combinations of alternatives and states of nature.
2. **Maximax** – optimistic: selects the alternative with the best conceivable outcome.
3. **Maximin (Wald)** – pessimistic: maximises the worst-case outcome.
4. **Laplace** – neutral: equal probabilities, highest average.
5. **Hurwicz** – flexible: α controls the degree of optimism.
6. **Savage-Niehans** – minimises maximum regret over missed opportunities.
7. **Expected value** – rational when probabilities are known.
8. Different methods may lead to **different recommendations**. The choice of method should reflect the decision makerś **risk attitude**.