Exercise Collection

Decision Tree – Rollback and Expected Values

English

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1 Level 1: Easy

Exercise 1.1 – Single-Stage Decision Tree

Exercise 1.1

A company is deciding whether to invest in a new project. If it invests, the profit (thousand euros) depends on market development:

Scenario	Probability	Profit
Good economy	0.5	200
Medium economy	0.3	80
Poor economy	0.2	−60

If it does not invest, the profit is a certain 0.

(a) Draw the decision tree.

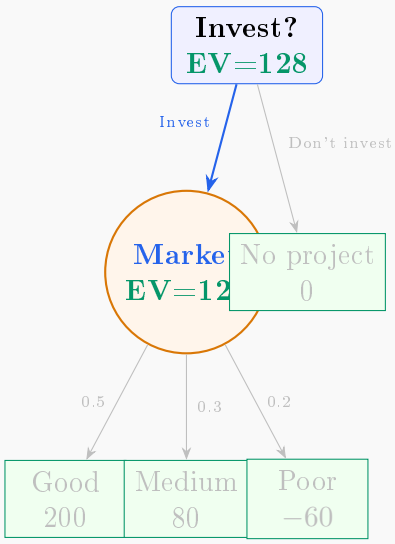
(b) Calculate the expected value at the chance node.

(c) Which decision does the rollback procedure recommend?

(d) Above which certain alternative profit would not investing be better?

Solution: Exercise 1.1

(a) Decision tree:



(b) Expected value:

$$\mathrm{EV}_{\text{Market}} = 0.5 \cdot 200 + 0.3 \cdot 80 + 0.2 \cdot (-60) = 100 + 24 - 12 = \mathbf{112}$$

Note: EV = 112, not 128. The tree diagram already shows the correct value.

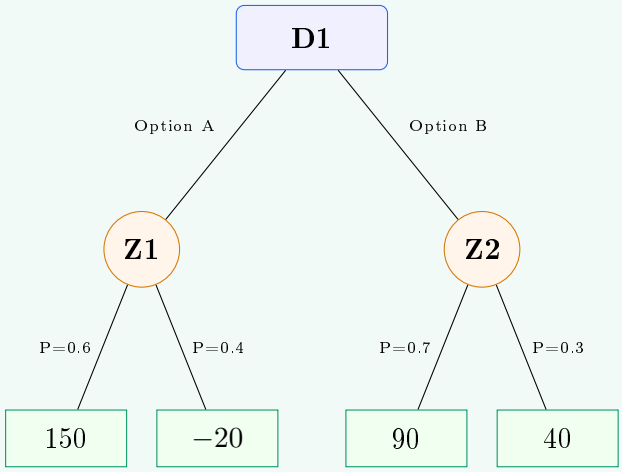
(c) Rollback: $\mathrm{EV}_{\text{root}} = \max(112,\ 0) = 112 \Rightarrow$ **Invest**.

(d) Threshold: Not investing is better if the certain alternative profit $g > 112$ thousand euros.
Optimal decision: Invest, EV = 112 thousand euros

Exercise 1.2 – Rollback: Calculation and Interpretation

Exercise 1.2

The following decision tree is given (payoffs in thousand euros):



(a) Calculate EV(Z1) and EV(Z2).

(b) Which option does decision node D1 choose?

(c) How large is the advantage of the optimal option (in thousand euros)?

Solution: Exercise 1.2

(a) Expected values:

$$\mathrm{EV}(Z_1) = 0.6 \cdot 150 + 0.4 \cdot (-20) = 90 - 8 = \mathbf{82}$$

$$\mathrm{EV}(Z_2) = 0.7 \cdot 90 + 0.3 \cdot 40 = 63 + 12 = \mathbf{75}$$

(b) Decision: $\mathrm{EV}(D_1) = \max(82,\ 75) = 82 \Rightarrow$ **Option A**.
(c) Advantage: $82 - 75 = \mathbf{7}$ thousand euros.

2 Level 2: Medium

Exercise 2.1 – Two-Stage Tree with Follow-up Decision

Exercise 2.1

A pharmaceutical company is evaluating the approval of a drug.
Stage 1: Conduct a clinical trial (cost 100) or cancel (payoff 0).
Stage 2 (after the trial):

- Trial success ($P = 0.7$): decision – market launch or license sale.
- Trial failure ($P = 0.3$): payoff -100.

Stage 3 (after success):

- **Market launch** (net of costs): Major success ($P = 0.5$, $+400$), Minor success ($P = 0.3$, $+150$), Failure ($P = 0.2$, -80).
- **License sale**: certain proceeds of $+120$.

(a) Perform the complete rollback procedure.

(b) Which strategy is optimal?

(c) How does the decision change if the trial's success probability drops to $P = 0.5$?

Solution: Exercise 2.1

(a) Rollback:
Chance node market launch:

$$\text{EV}_{\text{Market}} = 0.5 \cdot 400 + 0.3 \cdot 150 + 0.2 \cdot (-80) = 200 + 45 - 16 = \mathbf{229}$$

Decision node stage 3:

$$\text{EV}_{\text{Stage3}} = \max(229,\ 120) = \mathbf{229} \quad \Rightarrow \text{Market launch}$$

Chance node trial:

$$\text{EV}_{\text{Trial}} = 0.7 \cdot 229 + 0.3 \cdot (-100) = 160.3 - 30 = \mathbf{130.3}$$

Decision node stage 1:

$$\text{EV}_{\text{root}} = \max(130.3,\ 0) = \mathbf{130.3} \quad \Rightarrow \text{Conduct trial}$$

(b) Optimal strategy:

1. Conduct the trial.
2. On success: choose market launch (EV=229 $>$ license=120).
3. On failure: loss of 100.

Total EV: **130.3 thousand euros**.

(c) Sensitivity at $P(\text{success}) = 0.5$:

$$\text{EV}'_{\text{Trial}} = 0.5 \cdot 229 + 0.5 \cdot (-100) = 114.5 - 50 = \mathbf{64.5}$$

Still $> 0 \Rightarrow$ the trial remains recommendable, but noticeably less attractive.
Threshold: EV $= 0$ at p^*: $p \cdot 229 + (1-p) \cdot (-100) = 0 \Rightarrow 329p = 100 \Rightarrow p^* \approx 0.304$.
Only below a success probability of 30% should the trial be cancelled.

Exercise 2.2 – Comparison of Two Strategy Paths

Exercise 2.2

A logistics company is choosing between two expansion strategies.
Strategy A – Own fleet: High demand ($P = 0.6$, profit 300), Low demand ($P = 0.4$, loss -80).
Strategy B – Outsourcing: First contract available ($P = 0.8$):

- Contract A: Good partner ($P = 0.6$, profit 200), Poor partner ($P = 0.4$, profit 60).
- Contract B (alternative): certain profit of 100.

No contract available ($P = 0.2$): profit -20.

(a) Draw the complete decision tree.

(b) Perform the rollback.

(c) Which strategy is optimal?

Solution: Exercise 2.2

(a) Tree:
Strategy A is a simple chance node. Strategy B has an outer chance node (is a contract available?) and – if so – a decision node (contract A or B), followed by a further chance node under contract A.

(b) Rollback:
Strategy A:

$$\text{EV}_A = 0.6 \cdot 300 + 0.4 \cdot (-80) = 180 - 32 = \mathbf{148}$$

Strategy B – Contract A (chance node):

$$\text{EV}_{\text{ContrA}} = 0.6 \cdot 200 + 0.4 \cdot 60 = 120 + 24 = \mathbf{144}$$

Strategy B – Decision node (Contract A vs. B):

$$\text{EV}_{\text{Choice}} = \max(144,\ 100) = \mathbf{144} \quad \Rightarrow \text{Contract A}$$

Strategy B – Outer chance node:

$$\text{EV}_B = 0.8 \cdot 144 + 0.2 \cdot (-20) = 115.2 - 4 = \mathbf{111.2}$$

(c) Decision: $\max(\mathrm{EV}_A, \mathrm{EV}_B) = \max(148,\ 111.2) = 148 \Rightarrow$ **Strategy A (own fleet)**.

3 Level 3: Hard

Exercise 3.1 – Three-Stage Tree with Sensitivity Analysis

Exercise 3.1

A technology company is planning the development of a new AI platform.
Stage 1: Start research phase (cost 200) or cancel the project (0).
Stage 2 (after research): Technological breakthrough ($P = 0.5$) or partial success ($P = 0.5$).
Stage 3a (breakthrough): Market decision:

- Own platform: Market leadership ($P = 0.4$, $+800$), market share ($P = 0.4$, $+300$), flop ($P = 0.2$, -150).
- Cooperation with a large corporation: certain $+350$.

Stage 3b (partial success): Decision:

- Continue development (cost 100): renewed breakthrough ($P = 0.6$, $+400$), failure ($P = 0.4$, -100).
- License: certain $+80$.

(a) Perform the complete rollback.

(b) Identify the optimal strategy.

(c) What value must the certain cooperation profit take so that cooperation is chosen over the own platform?

(d) At which breakthrough probability p (stage 2) is the research phase just barely still advisable?

Solution: Exercise 3.1

(a) Rollback:
Stage 3a – Own platform:

$$\text{EV}_{\text{own}} = 0.4 \cdot 800 + 0.4 \cdot 300 + 0.2 \cdot (-150) = 320 + 120 - 30 = \mathbf{410}$$

Stage 3a – Decision at breakthrough:

$$\text{EV}_{3a} = \max(410,\ 350) = \mathbf{410} \quad \Rightarrow \text{Own platform}$$

Stage 3b – Continue development (net of cost 100):

$$\text{EV}_{\text{CD}} = 0.6 \cdot 400 + 0.4 \cdot (-100) - 100 = 240 - 40 - 100 = \mathbf{100}$$

Stage 3b – Decision at partial success:

$$\text{EV}_{3b} = \max(100,\ 80) = \mathbf{100} \quad \Rightarrow \text{Continue development}$$

Stage 2 – Chance node (net of research cost 200):

$$\mathrm{EV_R} = 0.5 \cdot 410 + 0.5 \cdot 100 - 200 = 205 + 50 - 200 = \mathbf{55}$$

Stage 1 – Decision node:

$$\mathrm{EV_{root}} = \max(55,\ 0) = \mathbf{55} \quad \Rightarrow \text{Do research}$$

(b) Optimal strategy:

1. Start the research phase.
2. On breakthrough: develop the own platform.
3. On partial success: continue development (do not license).

Total EV: **55 million euros**.
(c) Threshold for cooperation: Cooperation is better than in-house development if $g_{\text{Coop}} > 410$. Answer: the cooperation profit would need to exceed **410** million euros.
(d) Threshold for p: With general breakthrough probability p:

$$\mathrm{EV}_R(p) = p \cdot 410 + (1-p) \cdot 100 - 200 = 310p + 100 - 200 = 310p - 100$$

$\mathrm{EV}_R \geq 0 \Leftrightarrow 310p \geq 100 \Leftrightarrow p \geq \frac{100}{310} \approx \mathbf{0.323}$
Research pays off from a breakthrough probability of about **32%**.

Exercise 3.2 – Sequential Decision with Value of Information

Exercise 3.2

A real estate developer is considering buying a plot of land (cost 500). Afterward, they decide on the type of development.
After the purchase – zoning rights (chance node):

- Commercial permitted ($P = 0.6$): decision – commercial or residential.
- Residential only ($P = 0.4$): only residential development possible.

If commercial is permitted:

- **Commercial:** Good market ($P = 0.5$, +1200), poor market ($P = 0.5$, +200).
- **Residential:** Good market ($P = 0.7$, +900), poor market ($P = 0.3$, +400).

Residential only possible: Good market ($P = 0.7$, +900), poor market ($P = 0.3$, +400).
All amounts in thousand euros, net of purchase price.

(a) Perform the complete rollback.

(b) Should the plot of land be purchased?

(c) **Value of information:** An expert can clarify the zoning rights in advance (cost: X). Calculate the maximum price X the report is worth.

(d) How does the decision change if the good market for commercial occurs with only $P = 0.3$?

Solution: Exercise 3.2

(a) Rollback:
Residential development (identical in both cases):

$$\text{EV}_{\text{Res}} = 0.7 \cdot 900 + 0.3 \cdot 400 = 630 + 120 = \mathbf{750}$$

Commercial development:

$$\text{EV}_{\text{Com}} = 0.5 \cdot 1200 + 0.5 \cdot 200 = 600 + 100 = \mathbf{700}$$

Decision if commercial is permitted:

$$\text{EV}_{\text{Com-perm}} = \max(700,\ 750) = \mathbf{750} \quad \Rightarrow \text{Residential}$$

Note: Even if commercial is permitted, residential is more profitable!
Chance node zoning rights:

$$\text{EV}_{\text{Zoning}} = 0.6 \cdot 750 + 0.4 \cdot 750 = \mathbf{750}$$

(Since both cases yield the same EV.)
Decision node purchase: $\text{EV}_{\text{Purchase}} = 750 - 500 = 250$ (purchase price already deducted). $\max(250,\ 0) = \mathbf{250} \Rightarrow$ **Buy!**
(b) Purchase recommendation: Yes, EV = **250 thousand euros**.
(c) Value of information from the report:
With perfect information (expert clarifies zoning rights in advance):

- If commercial permitted ($P = 0.6$): choose residential (EV=750).
- If residential only ($P = 0.4$): residential (EV=750).

Since both cases yield the same EV, the information provides *no added value* regarding the development decision. EV with information = 750.
However: without information one buys for 500 and obtains EV=750, i.e., a net profit of 250. With information one could decide differently in the "residential only" case – but since residential is always chosen anyway, the value of the report is:
EVPI = EV with info − EV without info = $750 - 750 = \mathbf{0}$
In this case, the report has **no added value** – the development decision is the same regardless of the zoning rights.
(d) Sensitivity: P(**good market commercial**) = 0.3**:**

$$\text{EV}'_{\text{Com}} = 0.3 \cdot 1200 + 0.7 \cdot 200 = 360 + 140 = \mathbf{500}$$

Decision if commercial permitted: $\max(500,\ 750) = 750 \Rightarrow$ still residential. The overall decision does not change – residential dominates even more strongly.