Exercise Collection

Gold Swap Calculator

Exercises for Students

English

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1 Introduction

This exercise collection accompanies the **Gold Swap Documentation** and the **Gold Swap Calculator**. It contains:

- **Part A:** Conceptual questions (theory)
- **Part B:** Calculation exercises (by hand)
- **Part C:** Calculator exercises (practical)
- **Part D:** Case studies (application-oriented)
- **Part E:** Discussion questions (critical thinking)
- **Appendix:** Model solutions

1.1 Difficulty Levels

The exercises are labelled as follows:

- **Easy:** Basic concepts and simple calculations
- **Medium:** More complex calculations and analyses
- **Hard:** Advanced concepts and extensive calculations

1.2 Time Required

Estimated time for the entire exercise collection: **12–17 hours**

2 Part A: Conceptual Questions

2.1 Exercise A.1: Basic Concepts *(Easy)*

a) What is the difference between a gold swap and a standard interest rate swap?

b) Why are lease rates used instead of interest rates for the gold leg?

c) What does the "Pay EUR" direction mean?

d) Explain the difference between $\mathrm{DF}(t)$ and $L(t)$.

2.2 Exercise A.2: Valuation Formulas *(Medium)*

a) Derive the formula for the present value of the EUR leg:

$$\mathrm{PV}_{\mathrm{EUR}} = \sum_i \mathrm{CF}_{\mathrm{EUR}}(t_i) \times \mathrm{DF}_{\mathrm{EUR}}(t_i)$$

b) Why does the gold leg depend on TWO risk factors (lease rate AND gold spot)?

c) What happens to the net PV if:

- EUR zero rates rise?
- Gold lease rates rise?
- The gold spot price rises?

2.3 Exercise A.3: Sensitivities *(Medium)*

Consider a gold swap (Pay EUR) with the following sensitivities:

$$\frac{\partial \mathrm{PV}}{\partial \mathrm{DF}(1Y)} = -1{,}000{,}000 \text{ EUR}$$
$$\frac{\partial \mathrm{PV}}{\partial L(1Y)} = 1{,}500{,}000 \text{ EUR}$$
$$\frac{\partial \mathrm{PV}}{\partial S_{\mathrm{Gold}}} = 2{,}500 \text{ oz}$$

a) What does the negative EUR DF sensitivity mean?

b) Which risk factor has the largest exposure?

c) How does the PV change if:

- the EUR zero rate (1Y) rises by 10 bps?
- the gold lease rate (1Y) falls by 50 bps?

- gold spot falls by 50 EUR/oz?

Hint: Use the first derivative as an approximation.

2.4 Exercise A.4: VaR Concept *(Medium)*

a) Explain in your own words what VaR (99%, 1d) = 250,000 EUR means.

b) Why is VaR (99%, 10d) $\neq 10 \times$ VaR (99%, 1d)?

c) What is the difference between VaR and expected shortfall?

d) Name three limitations of the parametric VaR method.

2.5 Exercise A.5: Covariance Matrix *(Hard)*

a) Why are log returns used instead of absolute changes for the covariance matrix?

b) What does a negative covariance between two risk factors mean?

c) Explain why the portfolio variance is NOT simply the sum of the individual variances.

d) Why is the number of observations (251 vs. 3) critical for the VaR calculation?

3 Part B: Calculation Exercises

3.1 Exercise B.1: Discount Factor Calculation *(Easy)*

Given:

- EUR zero rate (6M) = 3.5%
- EUR zero rate (1Y) = 4.0%

a) Calculate $\text{DF}(0.5Y)$ and $\text{DF}(1.0Y)$.

b) What is the present value of 100,000 EUR in 6 months?

c) What is the present value of 100,000 EUR in 1 year?

3.2 Exercise B.2: Cash Flow Calculation *(Easy)*

Given a gold swap with:

- EUR notional: 2,000,000 EUR
- Gold notional: 1,500 oz
- EUR fixed rate: 2.5% p.a.
- Gold spot: 1,400 EUR/oz
- Gold lease (6M): 0.8%
- Payment frequency: semi-annual

a) Calculate $\text{CF}_{\text{EUR}}(0.5Y)$.

b) Calculate $\text{CF}_{\text{Gold}}(0.5Y)$.

c) Calculate the net cash flow difference at $t = 0.5Y$.

3.3 Exercise B.3: Full Valuation *(Medium)*

Given a 1-year gold swap (Pay EUR) with:

- EUR notional: 3,000,000 EUR
- Gold notional: 2,000 oz
- EUR fixed rate: 3.0% p.a.
- Gold spot: 1,500 EUR/oz
- EUR zero rate (6M): 3.2%
- EUR zero rate (1Y): 3.5%

- Gold lease (6M): 1.0%
- Gold lease (1Y): 0.8%
- Payment frequency: semi-annual

a) Calculate all cash flows (EUR and gold leg).

b) Calculate the discount factors.

c) Calculate PV_{EUR}, PV_{Gold} and the net PV.

d) Interpret the result: is the swap "in the money" or "out of the money"?

3.4 Exercise B.4: Sensitivity Calculation *(Medium)*

Use the data from Exercise B.3.

a) Calculate $\frac{\partial \text{PV}}{\partial \text{DF}(0.5Y)}$ and $\frac{\partial \text{PV}}{\partial \text{DF}(1.0Y)}$.

b) Calculate $\frac{\partial \text{PV}}{\partial L(0.5Y)}$ and $\frac{\partial \text{PV}}{\partial L(1.0Y)}$.

c) Calculate $\frac{\partial \text{PV}}{\partial S_{\text{Gold}}}$ (in oz).

d) Which risk factor dominates the total risk?

3.5 Exercise B.5: Portfolio Variance *(Hard)*

Given:

Sensitivity vector:

$$\mathbf{w} = \begin{bmatrix} -50{,}000 \\ -1{,}000{,}000 \\ 1{,}200{,}000 \\ 1{,}100{,}000 \\ 2{,}000 \end{bmatrix}$$

Covariance matrix (diagonal only, all covariances = 0 for simplicity):

$$\begin{aligned} \sigma^2(\text{DF } 0.5Y) &= 1 \times 10^{-8} \\ \sigma^2(\text{DF } 1.0Y) &= 4 \times 10^{-8} \\ \sigma^2(\text{Lease } 0.5Y) &= 3 \times 10^{-3} \\ \sigma^2(\text{Lease } 1.0Y) &= 4 \times 10^{-3} \\ \sigma^2(\text{Gold}) &= 2 \times 10^{-4} \end{aligned}$$

a) Calculate the portfolio variance (diagonal elements only).

b) Calculate the portfolio volatility.

c) Calculate VaR (99%, 1d) with $z = 2.326$.

d) Which risk factor contributes most to the total variance?

3.6 Exercise B.6: VaR Scaling *(Medium)*

Given: VaR (99%, 1d) = 150,000 EUR

a) Calculate VaR (99%, 10d) under the square-root-of-time assumption.

b) Calculate VaR (95%, 1d) if $z_{0.95} = 1.645$.

c) What is the expected shortfall at 99% if returns are normally distributed?

4 Part C: Calculator Exercises

4.1 Exercise C.1: Basic Operation *(Easy)*

Goal: Become familiar with the calculator

a) Load the file `gold_swap_timeseries_251_tage.csv`.

- How many rows were loaded?
- What is the time period?
- What is the last gold spot price?

b) Enter the following swap parameters:

- EUR notional: 10,000,000
- Gold notional: 5,000
- EUR fixed rate: 2.5%
- Maturity: 1 year
- Direction: Pay EUR

c) Record:

- Net PV
- All 5 sensitivities
- VaR (99%, 1d)

4.2 Exercise C.2: Direction Comparison *(Easy)*

Goal: Understand the effect of the direction

a) Calculate the swap with direction = "Pay EUR".

- Net PV = ?
- $\frac{\partial \text{PV}}{\partial \text{DF}(1Y)} = ?$

b) Change only the direction to "Receive EUR".

- Net PV = ?
- $\frac{\partial \text{PV}}{\partial \text{DF}(1Y)} = ?$

c) What do you observe? Why?

4.3 Exercise C.3: Sensitivity Analysis *(Medium)*

Goal: Understand the risk factors
Use the standard parameters (as in the documentation).

a) Record the base sensitivities.

b) Double the EUR notional (to 10 million).

- How do the EUR DF sensitivities change?
- How do the lease sensitivities change?
- Why?

c) Double the gold notional (to 6,000 oz).

- How do the lease sensitivities change?
- How does the gold spot sensitivity change?
- Why?

d) What is the dominant risk factor?

4.4 Exercise C.4: VaR Driver Analysis *(Medium)*

Goal: Identify the main risks

a) Calculate the standard swap and record VaR (99%, 1d).

b) Interpret: if the lease sensitivities were zero, how would the VaR change?

c) Interpret: if the DF sensitivities were zero, how would the VaR change?

d) Which type of risk factor dominates the VaR?

4.5 Exercise C.5: Time Series Length *(Hard)*

Goal: Understand the influence of the data history

a) Use the full 251-day time series.

- VaR (99%, 1d) = ?
- Portfolio volatility = ?

b) Discuss: what would happen with only 50 days?

c) Discuss: what would happen with only 10 days?

d) Discuss:

- How stable is the VaR?
- Why are 250 observations better than 10?
- What is the minimum for robust estimates?

4.6 Exercise C.6: Stress Scenario *(Hard)*

Goal: Understand extreme risks

a) Calculate VaR (99%, 1d) for the standard parameters.

b) Simulate a "lease rate shock" scenario:

- All lease rates rise by 200 bps (2%)
- What happens to the net PV?
- How large is the loss/gain?

c) Simulate a "gold crash" scenario:

- Gold spot falls by 30% (to ∼910 EUR/oz)
- What happens to the net PV?
- How large is the loss/gain?

d) Which scenario is worse? Why?
Hint: Use the sensitivities as a first approximation.

5 Part D: Case Studies

5.1 Case Study D.1: Hedging Decision *(Medium)*

Situation:

A gold mining company has an existing gold swap (Receive EUR, Pay Gold) with:

- EUR notional: 5,000,000 EUR
- Gold notional: 3,000 oz
- Remaining maturity: 1 year
- Current net PV: +500,000 EUR (in the money)

The VaR analysis shows:

- VaR (99%, 1d): 350,000 EUR
- Main risk: lease rate volatility

Questions:

a) What does a positive net PV mean for the company?

b) The risk management team proposes hedging the lease rate exposure. How could this work?

c) Should the company close the swap early while the PV is positive? Discuss the pros and cons.

d) What margin requirements might the counterparty impose?

5.2 Case Study D.2: Portfolio Optimisation *(Hard)*

Situation:

A bank holds a portfolio of 5 gold swaps with different parameters:

Swap	EUR Not.	Gold Not.	Net PV	VaR (99%, 1d)
A	5m	3,000 oz	−1.1m	350k
B	3m	2,000 oz	+0.5m	200k
C	10m	6,000 oz	−2.0m	700k
D	2m	1,500 oz	+0.2m	150k
E	8m	5,000 oz	−1.5m	600k

Questions:

a) What is the total net PV of the portfolio?

b) Is the portfolio VaR simply the sum of the individual VaRs? Why (not)?

c) Assume the correlation between the swaps is 0.7. Give a rough estimate of the portfolio VaR.

d) The bank wants to reduce its lease rate risk. Which swaps should be closed?

5.3 Case Study D.3: Regulatory Requirements *(Hard)*

Situation:

A hedge fund uses gold swaps for trading. The regulators require:

- VaR (99%, 10d) must not exceed 5% of equity capital
- Equity capital: 50 million EUR

Current portfolio:

- 10 gold swaps with aggregated VaR (99%, 1d) = 1.2 million EUR
- Average correlation: 0.6

Questions:

a) Calculate VaR (99%, 10d) under square-root-of-time.

b) Is the regulatory limit met (5% of 50 million = 2.5 million)?

c) If not, by how much must the position be reduced?

d) What alternatives are there apart from reducing the position?

5.4 Case Study D.4: Central Bank FX Liquidity *(Hard)*

Situation:

The central bank of an emerging market is facing an acute liquidity crisis:

- Gold reserves: 50 tonnes (current value: ∼2.1 billion EUR at 1,300 EUR/oz)
- Foreign exchange reserves: very low, only 500 million USD left
- Required: 2 billion USD for import financing and balance-of-payments stabilisation
- Political constraint: the government prohibits selling the gold reserves (national symbol)
- Alternative: international loan at 5% p.a. or an IMF programme (politically unpopular)

The central bank's finance department proposes a gold swap:

- Swap counterparty: large international bank
- Maturity: 1 year

- Gold remains the legal property of the central bank
- Lease rate: 0.8% p.a.
- Consideration: 2 billion USD (corresponds to ∼30 tonnes of gold)

Questions:

a) How does this gold swap work mechanically? Describe the cash flows at inception and at maturity.

b) Compare the costs:

- Gold swap (lease rate 0.8%)
- USD loan (5.0% interest)
- Gold sale + buy-back after 1 year

c) What risks arise for the central bank?

- The gold price rises by 20%?
- The gold price falls by 20%?
- The USD/EUR exchange rate changes?
- Counterparty risk?

d) Why is this gold swap politically sensitive?

- How is it reported on the balance sheet?
- Is the gold really still "there"?
- What happens in the event of default?
- Comparison with an IMF programme

e) Research: which countries have used gold swaps to raise foreign currency in the last 10 years?

6 Part E: Discussion Questions

6.1 Question E.1: Model Risk

a) The parametric VaR method assumes a normal distribution. What are the consequences if returns have "fat tails"?

b) How could model risk be quantified or reduced?

6.2 Question E.2: Lease Rate vs. Interest Rate

a) Why are gold lease rates typically lower than EUR zero rates?

b) What would happen if lease rates suddenly became higher than EUR rates?

c) Are there historical examples of extreme lease rate movements?

6.3 Question E.3: Practical Limitations

a) The documentation uses 251 historical observations. What are the pros and cons of more/less data?

b) How often should the covariance matrix be updated (daily, weekly, monthly)?

c) What do you do if there are no liquid lease rate quotes?

6.4 Question E.4: Alternative Methods

a) Compare parametric VaR, historical simulation and Monte Carlo. What are the pros and cons?

b) When would you prefer historical simulation?

c) When would you prefer Monte Carlo?

6.5 Question E.5: Business Sense

a) Why would a company enter into a gold swap at all?

b) Give 4 concrete examples of institutions/companies that need gold swaps (including central banks).

c) Is a gold swap a hedging instrument or a speculative instrument? Discuss.

Appendix: Model Solutions

Part A: Conceptual Questions – Solutions

Solution A.1:

a) Gold swap vs. IRS:

- **IRS:** Both legs are based on interest rates (floating vs. fixed)
- **Gold swap:** One leg is based on gold (lease rate + gold spot), the other on an EUR interest rate
- A gold swap carries additional commodity risk

b) Why lease rates:

- Lease rates are the "cost" of borrowing gold
- They represent the interest for borrowing physical gold
- Analogous to repo rates for bonds

c) "Pay EUR" means:

- The bank pays the EUR leg (based on the EUR fixed rate)
- The bank receives the gold leg (based on lease rate + gold spot)
- The direction determines the sign of the net PV

d) DF(t) vs. $L(t)$:

- **DF(t):** Discount factor, derived from the EUR zero rate, used to discount cash flows
- **L(t):** Lease rate, the "interest rate" for borrowing gold, determines the gold leg cash flows
- DF(t) < 1, $L(t)$ is a percentage (typically 0.1% – 1%)

Solution A.2:

a) Derivation of PV_{EUR}:

Each cash flow $\text{CF}_{\text{EUR}}(t_i)$ has a future value. To obtain the present value, discount with $\text{DF}_{\text{EUR}}(t_i)$. Sum over all payment dates:

$$\text{PV}_{\text{EUR}} = \text{CF}_{\text{EUR}}(t_1) \times \text{DF}(t_1) + \text{CF}_{\text{EUR}}(t_2) \times \text{DF}(t_2) + \ldots = \sum_i \text{CF}_{\text{EUR}}(t_i) \times \text{DF}_{\text{EUR}}(t_i)$$

b) Two risk factors for the gold leg:

- **Lease rate $L(t)$:** Determines the interest payment on the gold value
- **Gold spot S_{Gold}:** Determines the value of the gold notional

- Cash flow = $S_{\text{Gold}} \times N_{\text{oz}} \times [L(t) \times \Delta t + 1 \text{ at the end}]$
- Both factors can move independently

c) Effects (Pay EUR):

- **EUR rates rise → DF falls → $\mathbf{PV_{EUR}}$ falls → net PV rises** (better for Pay EUR)
- **Lease rates rise → $\mathbf{CF_{Gold}}$ rises → $\mathbf{PV_{Gold}}$ rises → net PV rises** (better for Pay EUR)
- **Gold spot rises → $\mathbf{CF_{Gold}}$ rises → $\mathbf{PV_{Gold}}$ rises → net PV rises** (better for Pay EUR)

Solution A.3:

a) Negative EUR DF sensitivity:

- $\frac{\partial \text{PV}}{\partial \text{DF}(1Y)} = -1{,}000{,}000 \text{ EUR} < 0$
- If the DF rises (EUR rate falls) → PV falls by 1 million EUR
- This means: the Pay EUR position is larger than the Receive Gold position at this tenor
- The EUR cash flow dominates

b) Largest exposure:

- Lease: 1,500,000 EUR (absolute)
- EUR DF: 1,000,000 EUR (absolute)
- Gold spot: 2,500 oz ≈ 2,500 × 1,300 = 3,250,000 EUR (if gold ≈ 1,300)
- **Gold spot has the largest exposure** (when converted to EUR)
- **But:** the lease rate is probably more volatile

c) PV changes:
EUR rate +10 bps:

$$\Delta\text{DF} \approx -\text{DF} \times t \times \Delta r = -0.977 \times 1 \times 0.001 \approx -0.00098$$
$$\Delta\text{PV} \approx -1{,}000{,}000 \times (-0.00098) \approx +980 \text{ EUR}$$

Lease rate −50 bps:

$$\Delta\text{PV} \approx 1{,}500{,}000 \times (-0.005) = -7{,}500 \text{ EUR}$$

Gold spot -50 EUR:

$$\Delta \text{PV} \approx 2{,}500 \times (-50) = -125{,}000 \text{ EUR}$$

The gold spot move has the largest effect!

Solution A.4:

a) VaR (99%, 1d) = 250,000 EUR means:

- "With 99% probability the swap loses **no more than** 250,000 EUR in one day"
- In other words: "On 99 out of 100 days the loss is $\leq$ 250,000 EUR"
- Or: "Only on 1 out of 100 days does the loss exceed 250,000 EUR"

b) Why VaR(10d) $\neq 10 \times$ VaR(1d):

- Returns scale with $\sqrt{\text{time}}$, not linearly
- VaR(10d) = VaR(1d) $\times\sqrt{10} \approx 3.16\times$ VaR(1d)
- Reason: volatility grows with the square root of time
- Diversification over time

c) VaR vs. expected shortfall (ES):

- **VaR:** Only the threshold (the maximum loss at 99%)
- **ES (CVaR):** Average loss IF the VaR is exceeded
- ES answers: "What happens in the worst 1% of cases?"
- ES is always $\geq$ VaR
- ES is coherent, VaR is not

d) Limitations of parametric VaR:

1. **Normal distribution assumption:** fat tails are underestimated
2. **Linear approximation:** delta-normal, no gamma effects
3. **Constant correlations:** they change in crises
4. **Historical data:** past $\neq$ future
5. **No worst-case information:** says nothing about tail losses

Solution A.5:

a) Log returns vs. absolute changes:
Log returns: $r = \ln(P_t/P_{t-1})$

- Time-additive: $r(2d) = r(1d) + r(1d)$
- Symmetric: $r(+10\%) \neq -r(-10\%)$, but the log scale is more symmetric
- Closer to a normal distribution

Absolute changes: $\Delta = P_t - P_{t-1}$

- Not well suited for percentage risks
- Price-dependent

b) Negative covariance:

- $\text{Cov}(X, Y) < 0$ means: X and Y move in opposite directions
- When X rises, Y tends to fall
- Example: EUR DF and EUR rate (when the rate rises, the DF falls)
- **Diversification effect:** the portfolio variance becomes smaller

c) Portfolio variance $\neq$ sum of variances:

$$\begin{aligned}\text{Var}(w_1X_1 + w_2X_2) &= w_1^2\text{Var}(X_1) + w_2^2\text{Var}(X_2) + 2w_1w_2\text{Cov}(X_1, X_2) \\ &\neq w_1^2\text{Var}(X_1) + w_2^2\text{Var}(X_2) \quad [\text{if Cov} \neq 0]\end{aligned}$$

- The covariance term can be positive or negative
- Negative correlation reduces total risk (diversification)
- Positive correlation increases total risk (concentration)

d) Number of observations:
3 observations: only 2 returns $\rightarrow$ extremely uncertain covariance estimate

- Very large standard error
- Few degrees of freedom

251 observations: 250 returns $\rightarrow$ robust estimate

- Smaller standard error $\propto 1/\sqrt{n}$
- More degrees of freedom
- Rule of thumb: at least 180–250 for a covariance matrix

Part B: Calculation Exercises – Solutions

Solution B.1:

a) Discount factors:

$$\text{DF}(0.5Y) = e^{-0.035\times 0.5} = e^{-0.0175} = 0.98266$$
$$\text{DF}(1.0Y) = e^{-0.040\times 1.0} = e^{-0.040} = 0.96079$$

b) PV of 100,000 EUR in 6M:

$$\text{PV} = 100{,}000 \times 0.98266 = 98{,}266 \text{ EUR}$$

c) PV of 100,000 EUR in 1Y:

$$\text{PV} = 100{,}000 \times 0.96079 = 96{,}079 \text{ EUR}$$

Solution B.2:

a) $\text{CF}_{\text{EUR}}(0.5Y)$:

$$\text{CF}_{\text{EUR}}(0.5Y) = N_{\text{EUR}} \times r_{\text{EUR}} \times \Delta t = 2{,}000{,}000 \times 0.025 \times 0.5 = 25{,}000 \text{ EUR}$$

b) $\text{CF}_{\text{Gold}}(0.5Y)$:

$$V_{\text{Gold}} = S_{\text{Gold}} \times N_{\text{oz}} = 1{,}400 \times 1{,}500 = 2{,}100{,}000 \text{ EUR}$$
$$\begin{aligned}\text{CF}_{\text{Gold}}(0.5Y) &= V_{\text{Gold}} \times L(0.5Y) \times \Delta t \\ &= 2{,}100{,}000 \times 0.008 \times 0.5 = 8{,}400 \text{ EUR}\end{aligned}$$

c) Net cash flow (at $t = 0.5Y$, Pay EUR):

$$\text{Net CF} = \text{CF}_{\text{Gold}} - \text{CF}_{\text{EUR}} = 8{,}400 - 25{,}000 = -16{,}600 \text{ EUR}$$

Negative $\rightarrow$ pays more than it receives

Solution B.3:

a) All cash flows:
EUR leg:

$$\text{CF}_{\text{EUR}}(0.5Y) = 3{,}000{,}000 \times 0.03 \times 0.5 = 45{,}000 \text{ EUR}$$
$$\text{CF}_{\text{EUR}}(1.0Y) = 45{,}000 + 3{,}000{,}000 = 3{,}045{,}000 \text{ EUR}$$

Gold leg:

$$\begin{aligned} V_{\text{Gold}} &= 1{,}500 \times 2{,}000 = 3{,}000{,}000 \text{ EUR} \\ \text{CF}_{\text{Gold}}(0.5Y) &= 3{,}000{,}000 \times 0.01 \times 0.5 = 15{,}000 \text{ EUR} \\ \text{CF}_{\text{Gold}}(1.0Y) &= 3{,}000{,}000 \times 0.008 \times 0.5 + 3{,}000{,}000 \\ &= 12{,}000 + 3{,}000{,}000 = 3{,}012{,}000 \text{ EUR} \end{aligned}$$

b) Discount factors:

$$\begin{aligned} \text{DF}(0.5Y) &= e^{-0.032 \times 0.5} = e^{-0.016} = 0.98413 \\ \text{DF}(1.0Y) &= e^{-0.035 \times 1.0} = e^{-0.035} = 0.96562 \end{aligned}$$

c) Present values:

$$\begin{aligned} \text{PV}_{\text{EUR}} &= 45{,}000 \times 0.98413 + 3{,}045{,}000 \times 0.96562 \\ &= 44{,}286 + 2{,}940{,}313 = 2{,}984{,}599 \text{ EUR} \end{aligned}$$

$$\begin{aligned} \text{PV}_{\text{Gold}} &= 15{,}000 \times 0.98413 + 3{,}012{,}000 \times 0.96562 \\ &= 14{,}762 + 2{,}908{,}497 = 2{,}923{,}259 \text{ EUR} \end{aligned}$$

$$\begin{aligned} \text{Net PV (Pay EUR)} &= \text{PV}_{\text{Gold}} - \text{PV}_{\text{EUR}} \\ &= 2{,}923{,}259 - 2{,}984{,}599 = -61{,}340 \text{ EUR} \end{aligned}$$

d) Interpretation:

- Net PV $= -61{,}340$ EUR < 0
- **The swap is "OUT OF THE MONEY"** from the Pay EUR perspective
- The EUR leg is worth more than the gold leg
- If the swap were closed today, 61,340 EUR would have to be paid

Solution B.4:

Use the data from B.3.

a) EUR DF sensitivities:

$$\begin{aligned} \frac{\partial \text{PV}}{\partial \text{DF}(0.5Y)} &= \text{CF}_{\text{Gold}}(0.5Y) - \text{CF}_{\text{EUR}}(0.5Y) \\ &= 15{,}000 - 45{,}000 = -30{,}000 \text{ EUR} \end{aligned}$$

$$\begin{aligned} \frac{\partial \text{PV}}{\partial \text{DF}(1.0Y)} &= \text{CF}_{\text{Gold}}(1.0Y) - \text{CF}_{\text{EUR}}(1.0Y) \\ &= 3{,}012{,}000 - 3{,}045{,}000 = -33{,}000 \text{ EUR} \end{aligned}$$

b) Lease sensitivities:

$$\frac{\partial \text{PV}}{\partial L(0.5Y)} = S_{\text{Gold}} \times N_{\text{oz}} \times \Delta t \times \text{DF}(0.5Y)$$
$$= 1{,}500 \times 2{,}000 \times 0.5 \times 0.98413 = 1{,}476{,}195 \text{ EUR}$$

$$\frac{\partial \text{PV}}{\partial L(1.0Y)} = S_{\text{Gold}} \times N_{\text{oz}} \times \Delta t \times \text{DF}(1.0Y)$$
$$= 1{,}500 \times 2{,}000 \times 0.5 \times 0.96562 = 1{,}448{,}430 \text{ EUR}$$

c) Gold spot sensitivity:

$$\frac{\partial \text{PV}}{\partial S_{\text{Gold}}} = N_{\text{oz}} \times [L(0.5Y) \times \Delta t \times \text{DF}(0.5Y) + L(1.0Y) \times \Delta t \times \text{DF}(1.0Y) + \text{DF}(1.0Y)]$$
$$= 2{,}000 \times [0.01 \times 0.5 \times 0.98413 + 0.008 \times 0.5 \times 0.96562 + 0.96562]$$
$$= 2{,}000 \times [0.00492 + 0.00386 + 0.96562]$$
$$= 2{,}000 \times 0.97440 = 1{,}948.8 \text{ oz}$$

d) Dominant risk factor:

- EUR DF: ∼30k EUR (small)
- Lease: ∼1.45 million EUR (VERY LARGE!)
- Gold spot: ∼1,949 oz ≈ 2.9 million EUR

Lease rates dominate the risk!

Solution B.5:

a) Portfolio variance (diagonal only):

$$\sigma_P^2 = \sum_i w_i^2 \times \sigma_i^2$$
$$= (-50{,}000)^2 \times 1 \times 10^{-8} + (-1{,}000{,}000)^2 \times 4 \times 10^{-8}$$
$$+ (1{,}200{,}000)^2 \times 3 \times 10^{-3} + (1{,}100{,}000)^2 \times 4 \times 10^{-3}$$
$$+ (2{,}000)^2 \times 2 \times 10^{-4}$$
$$= 25 + 40{,}000 + 4{,}320{,}000{,}000 + 4{,}840{,}000{,}000 + 800$$
$$\approx 9{,}160{,}040{,}825 \text{ EUR}^2$$

b) Portfolio volatility:

$$\sigma_P = \sqrt{9{,}160{,}040{,}825} = 95{,}708 \text{ EUR}$$

c) VaR (99%, 1d):

$$\text{VaR} = 2.326 \times 95{,}708 = 222{,}617 \text{ EUR}$$

d) Contribution to variance:

$$\begin{aligned}\text{Lease } 0.5Y &: (1{,}200{,}000)^2 \times 3 \times 10^{-3} = 4{,}320{,}000{,}000 \quad (47\%)\\ \text{Lease } 1.0Y &: (1{,}100{,}000)^2 \times 4 \times 10^{-3} = 4{,}840{,}000{,}000 \quad (53\%)\\ \text{Rest} &: \text{negligible}\end{aligned}$$

Lease rates contribute 100% of the variance!

Solution B.6:

a) VaR (99%, 10d):

$$\text{VaR}(10d) = \text{VaR}(1d) \times \sqrt{10} = 150{,}000 \times 3.162 = 474{,}300 \text{ EUR}$$

b) VaR (95%, 1d):

$$\text{VaR}_{95} = \frac{z_{0.95}}{z_{0.99}} \times \text{VaR}_{99} = \frac{1.645}{2.326} \times 150{,}000 = 0.7073 \times 150{,}000 = 106{,}095 \text{ EUR}$$

c) Expected shortfall (99%):
For a normal distribution:

$$\begin{aligned}\text{ES}_{99} &= \sigma \times \frac{\phi(z_{0.99})}{1 - 0.99}\\ \sigma &= \frac{\text{VaR}_{99}}{z_{0.99}} = \frac{150{,}000}{2.326} = 64{,}496 \text{ EUR}\\ \phi(2.326) &\approx 0.0267\\ \text{ES}_{99} &= 64{,}496 \times \frac{0.0267}{0.01} \approx 172{,}000 \text{ EUR}\end{aligned}$$

Rule of thumb: ES $\approx 1.15\times$ VaR at 99%

Part C: Calculator Exercises – Solutions

Solution C.1:

a) Loading the CSV:

- Rows loaded: **251**
- Period: **01.01.2023 to 08.09.2023**
- Last gold spot: **1,302.51 EUR/oz**

b) Parameters entered ✓

c) Results (depend on the calculator implementation):

- Net PV: $\sim -2{,}296{,}232$ EUR (Pay EUR with the larger notional)
- Sensitivities:
  - $\frac{\partial \text{PV}}{\partial \text{DF}(0.5Y)}$: $\sim -63{,}841$ EUR
  - $\frac{\partial \text{PV}}{\partial \text{DF}(1.0Y)}$: $\sim -1{,}893{,}153$ EUR
  - $\frac{\partial \text{PV}}{\partial L(0.5Y)}$: $\sim 3{,}222{,}464$ EUR
  - $\frac{\partial \text{PV}}{\partial L(1.0Y)}$: $\sim 3{,}182{,}649$ EUR
  - $\frac{\partial \text{PV}}{\partial S_{\text{Gold}}}$: $\sim 4{,}910$ oz
- VaR (99%, 1d): $\sim 583{,}372$ EUR

Solution C.2:

a) Pay EUR:

- Net PV: e.g. $-1{,}148{,}116$ EUR (negative)
- $\frac{\partial \text{PV}}{\partial \text{DF}(1Y)}$: $-1{,}135{,}892$ EUR (negative)

b) Receive EUR (direction changed):

- Net PV: $+1{,}148{,}116$ EUR (positive, sign flipped!)
- $\frac{\partial \text{PV}}{\partial \text{DF}(1Y)}$: $+1{,}135{,}892$ EUR (positive, sign flipped!)

c) Observation:

- **ALL values only change their sign!**
- Net $\text{PV}_{\text{receive}} = -\text{Net PV}_{\text{pay}}$
- Reason: the direction completely reverses the perspective
- What was a payment before is now a receipt

Solution C.3:

a) Base sensitivities (standard 5M/3k):

$$\frac{\partial \text{PV}}{\partial \text{DF}(0.5Y)} : -38{,}305 \text{ EUR}$$
$$\frac{\partial \text{PV}}{\partial \text{DF}(1.0Y)} : -1{,}135{,}892 \text{ EUR}$$
$$\frac{\partial \text{PV}}{\partial L(0.5Y)} : 1{,}933{,}479 \text{ EUR}$$
$$\frac{\partial \text{PV}}{\partial L(1.0Y)} : 1{,}909{,}589 \text{ EUR}$$
$$\frac{\partial \text{PV}}{\partial S_{\text{Gold}}} : 2{,}946 \text{ oz}$$

b) EUR notional doubled (10M/3k):

- EUR DF sensitivities: ∼double (linear in N_{EUR}!)
  - $\frac{\partial \text{PV}}{\partial \text{DF}(0.5Y)}$: $\sim -76{,}610$ EUR
  - $\frac{\partial \text{PV}}{\partial \text{DF}(1.0Y)}$: $\sim -2{,}271{,}784$ EUR
- Lease sensitivities: UNCHANGED (they do not depend on N_{EUR})
  - $\frac{\partial \text{PV}}{\partial L(0.5Y)}$: 1,933,479 EUR (same!)
  - $\frac{\partial \text{PV}}{\partial L(1.0Y)}$: 1,909,589 EUR (same!)
- Why: lease sensitivity = $S_{\text{Gold}} \times N_{\text{oz}} \times \Delta t \times \text{DF}$ (no N_{EUR}!)

c) Gold notional doubled (5M/6k):

- EUR DF sensitivities: change (because CF_{Gold} rises)
- Lease sensitivities: ∼double (linear in N_{oz}!)
  - $\frac{\partial \text{PV}}{\partial L(0.5Y)}$: $\sim 3{,}866{,}958$ EUR
  - $\frac{\partial \text{PV}}{\partial L(1.0Y)}$: $\sim 3{,}819{,}178$ EUR
- Gold spot sensitivity: ∼doubled
  - $\frac{\partial \text{PV}}{\partial S_{\text{Gold}}}$: $\sim 5{,}892$ oz
- Why: gold sensitivity = $N_{\text{oz}} \times [\ldots]$

d) Dominant risk factor:

- **Lease rates!**
- Sensitivities ∼1.9 million EUR (very large)
- Lease volatility ∼6% daily (very high)
- The combination = dominant risk

Solution C.4–C.6:

Note: The solutions to C.4–C.6 depend strongly on the specific calculator implementation and on the students' experiments. These exercises are designed as practical exercises for exploration and analysis with the tool.

Key insights students should gain:

C.4 (VaR drivers):

- Lease rates dominate the VaR almost completely ($> 95\%$ of the risk)
- EUR DFs contribute only minimally
- Gold spot is relevant, but less than lease

C.5 (Time series length):

- 251 days: stable, robust
- 50 days: still usable, but more volatile
- 10 days: very unstable, not reliable
- Minimum: 180–250 observations

C.6 (Stress scenarios):

- Lease rate shock (+200 bps): gain $\sim$+77k EUR (Pay EUR benefits!)
- Gold crash (-30%): loss ~ -1.15 million EUR (very painful!)
- VaR captures normal volatility, not crash scenarios

Part D: Case Studies – Solutions

Solution D.1:

a) Positive net PV:

- +500,000 EUR in the money
- If closed today: the company receives 500k EUR
- But: this is the current value and may change

b) Lease rate hedge:
Options:

1. **Lease rate swap:** fixed vs. floating lease rate
2. **Gold futures:** indirect, since futures implicitly contain lease rates
3. **Options on lease rates:** expensive, but flexible
4. **Reduce the position:** close part of the swap

c) Close early?
Pro:

- Realise the 500k gain (certain)
- Eliminate the VaR risk of 350k
- No ongoing monitoring required

Contra:

- The operational hedging function is lost
- The original hedging need may still exist
- Transaction costs
- Further increases in value are missed

Recommendation: depends on:

- The original hedging purpose
- Risk appetite
- Expectations about future market movements

d) Margin requirements:

- With a positive PV: the counterparty has a loss
- Initial margin: ∼10–20% of the notional = 500k – 1M EUR
- Variation margin: daily mark-to-market
- VaR can serve as a basis: e.g. $2 \times \text{VaR}(10d) = 2 \times 1.1M = 2.2M$ EUR

Solution D.2:

a) Total net PV:

$$\text{PV}_{\text{total}} = -1.1 + 0.5 - 2.0 + 0.2 - 1.5 = -3.9 \text{ million EUR}$$

The bank has an aggregate loss of 3.9 million EUR

b) Portfolio VaR ≠ sum:

$$\text{VaR}_{\text{sum}} = 350 + 200 + 700 + 150 + 600 = 2{,}000\text{k EUR}$$

But: portfolio VaR < 2,000k (due to diversification)
Why:

- The swaps are not perfectly correlated
- The diversification effect reduces the total VaR
- Only with correlation = 1 would VaR = sum

c) Portfolio VaR with $\rho = 0.7$:
With a uniform correlation ρ: $\text{VaR}_{\text{Portfolio}} = \sqrt{\sum_i \text{VaR}_i^2 + \rho \sum_{i \neq j} \text{VaR}_i \, \text{VaR}_j}$

$$\text{VaR}_{\text{Portfolio}} = \sqrt{1{,}035{,}000 + 0.7 \times 2{,}965{,}000} \approx 1{,}764\text{k EUR} \; (\approx 0.88 \times \text{VaR}_{\text{sum}})$$

d) Which swaps to close:
Criteria:

1. **Largest individual VaR:** swap C (700k) → first priority
2. **Largest loss (PV):** swap C ($-2.0M$) → strengthens the argument
3. **Close C:** reduces VaR by ∼600k (from 1,764k to 1,160k, with diversification)

Alternative: close swap E (600k VaR, $-1.5M$ PV)
Recommendation: close swap C (worst risk/return profile)

Solution D.3:

a) VaR (99%, 10d):

$$\text{VaR}(10d) = \text{VaR}(1d) \times \sqrt{10} = 1.2 \text{ million} \times 3.162 = 3.794 \text{ million EUR}$$

b) Regulatory limit:

$$\begin{aligned} \text{Limit} &= 5\% \times 50 \text{ million} = 2.5 \text{ million EUR} \\ \text{Current VaR}(10d) &= 3.794 \text{ million EUR} \\ 3.794 > 2.5 \quad &\rightarrow \text{ LIMIT EXCEEDED!} \end{aligned}$$

c) Required reduction:

$$\begin{aligned} \text{Required VaR} &= 2.5 \text{ million} \\ \text{Current VaR} &= 3.794 \text{ million} \\ \text{Reduction factor} &= \frac{2.5}{3.794} = 0.659 = 65.9\% \\ &\rightarrow \text{close } 34.1\% \text{ of the position} \end{aligned}$$

d) Alternatives:

1. **Hedging:**
   - Lease rate swaps as an offset
   - Offsetting gold futures
   - Reduces VaR without closing the position
2. **Increase equity capital:**
   - 50 million $\rightarrow$ 76 million EUR
   - $5\% \times 76$ million $= 3.8$ million > 3.794 million ✓
3. **Diversification:**
   - Reduce the correlation between swaps
   - Different maturities, directions
4. **Risk reduction:**
   - Smaller notionals per swap
   - More, but smaller positions
5. **Change the VaR method:**
   - Historical simulation (if lower)
   - Monte Carlo with better tail modelling
   - But: regulators must approve

Solution D.4:

a) Mechanics of the gold swap:
At inception (day 0):

- Central bank: transfers 30 tonnes of gold to the swap counterparty (physically or legally)

- Swap counterparty: pays 2 billion USD to the central bank
- Legal ownership: remains with the central bank (important for the balance sheet!)
- Possession: temporarily with the swap counterparty

During the term (1 year):

- The central bank pays a "lease rate" of 0.8% p.a. on 2 billion USD
- = 16 million USD per year
- Possibly in quarterly payments

At maturity (day 365):

- Central bank: repays 2 billion USD
- Swap counterparty: returns the gold (30 tonnes)
- Plus the final lease rate payment

b) Cost comparison:
Option 1: Gold swap

$$\text{Cost} = 2{,}000{,}000{,}000 \times 0.008 = 16{,}000{,}000 \text{ USD}$$
$$\text{Cost in } \% = 0.8\% \text{ p.a.}$$

Option 2: USD loan

$$\text{Cost} = 2{,}000{,}000{,}000 \times 0.05 = 100{,}000{,}000 \text{ USD}$$
$$\text{Cost in } \% = 5.0\% \text{ p.a.}$$

Option 3: Gold sale + buy-back

- Today: sell 30t of gold for 2 billion USD
- In 1 year: buy the gold back
- **Risk:** the gold price may have risen!
- If gold +10%: buy-back costs 2.2 billion USD $\rightarrow$ loss of 200 million USD
- If gold −10%: buy-back costs 1.8 billion USD $\rightarrow$ gain of 200 million USD
- Plus: politically unacceptable ("selling the national reserves")

Conclusion:

- The gold swap is by far the cheapest (16 million vs. 100 million)
- 84% cost saving vs. the USD loan
- No gold price risk (unlike a sale)
- Politically acceptable (gold remains "owned")

c) Risks for the central bank:
Risk 1: gold price rises by 20%

- Gold now worth: 30t × 1,560 EUR/oz = 2.52 billion EUR (instead of 2.1 billion)
- Opportunity cost: 420 million EUR forgone
- **But:** no loss! The gold is returned at the old value
- Balance sheet effect: unrealised gain on the gold

Risk 2: gold price falls by 20%

- Gold now worth: 30t × 1,040 EUR/oz = 1.68 billion EUR
- Loss: 420 million EUR loss in value
- **But:** the same as if the gold had stayed in the reserves
- No additional risk from the swap

Risk 3: USD/EUR exchange rate

- If the USD weakens: USD debt is worth less (positive!)
- If the USD strengthens: USD debt is worth more (negative)
- Typical: USD debt for an emerging market = currency risk

Risk 4: counterparty risk

- The swap counterparty could default
- The gold is gone if the counterparty goes bankrupt
- **Mitigation:** only with AAA banks, collateral agreements
- Lehman 2008: many gold swaps were lost

Risk 5: roll-over risk

- After 1 year: the swap must be renewed or 2 billion USD repaid
- If the FX situation has not improved: a new swap is needed
- Lease rates may have risen
- Dependence on the swap counterparty

d) Political sensitivity:
Balance sheet reporting:

- Gold remains an asset (legal ownership)
- USD debt appears as a liability
- Net: the balance sheet deteriorates (debt rises)
- **But:** FX reserves also rise (USD cash)

Is the gold really still "there"?

- Legally: yes
- Physically: no (with the swap counterparty)
- **Problem:** the opposition could claim "the gold has been sold"
- The public often does not understand the difference
- Possible loss of confidence

In the event of default:

- If the central bank cannot repay the 2 billion
- The gold goes to the swap counterparty (collateral)
- National disaster ("gold lost")
- The government could fall

Comparison with an IMF programme:
IMF programme:

- Pro: low interest rates, long-term
- Contra: strict conditions (austerity, reforms)
- Contra: loss of sovereignty
- Contra: politically toxic ("foreign dependence")

Gold swap:

- Pro: no political conditions
- Pro: quickly available
- Pro: cheaper than commercial loans
- Contra: gold risk
- Contra: short-term (1 year), no long-term fix

Political assessment:

- Gold swap = "pragmatic solution"
- The government can say: "We keep the gold, we just use it cleverly"
- The opposition can say: "The gold was secretly sold off"
- Transparency is critical!

e) Historical examples (research):
Turkey (2020–2021):

- Massive gold swaps with commercial banks

- Volume: >10 billion USD
- FX reserves artificially inflated
- Criticism: lack of transparency

Venezuela (2018):

- Attempted gold swaps with Russia
- Sanctions made deals difficult
- Gold physically flown to Uganda

Ghana (2015):

- 1 billion USD gold swap with Citibank
- For budget financing
- Successfully repaid

India (1991):

- Balance-of-payments crisis
- 67 tonnes of gold pledged to the Bank of England
- 400 million USD loan
- Gold later recovered
- A national humiliation, but the crisis was overcome

Egypt (2016):

- Gold swaps for FX stabilisation
- Part of broader reform packages

Key lesson:

- Gold swaps are a common tool for emerging markets
- Often a sign of economic weakness
- Can work if managed transparently and professionally
- The risks must be understood
- Not a long-term solution, only a bridge

Part E: Discussion Questions – Answers

The answers to Part E are conceptual and are meant to stimulate discussion. There is often no single "correct" answer.

Answer E.1: Model Risk

a) Consequences of fat tails:
If returns have fat tails (e.g. a t-distribution instead of normal):

- **VaR is underestimated:** the normal distribution has tails that are too thin
- **Extreme events are more frequent:** the 1% quantile is larger in reality
- **A 99% VaR might actually be a 95% VaR:** more losses than expected

Example:

$$\begin{aligned} \text{Normal:} \quad & \text{VaR}(99\%) = 350\text{k} \rightarrow \text{exceeded 1\% of the time} \\ \text{Fat tail:} \quad & \text{VaR}(99\%) = 350\text{k} \rightarrow \text{exceeded 5\% of the time!} \end{aligned}$$

b) Reducing model risk:

1. **Backtesting:**
   - Compare predicted VaR with actual losses
   - Count VaR exceedances
   - Should be $\approx 1\%$; if not $\rightarrow$ adjust the model
2. **Alternative distributions:**
   - t-distribution (fat tails)
   - Generalized extreme value (GEV)
   - Empirical distribution
3. **Multiple methods:**
   - Parametric, historical and Monte Carlo in parallel
   - Compare the results
   - If they differ widely $\rightarrow$ be cautious
4. **Stress testing:**
   - Supplement VaR with worst-case scenarios
   - Expected shortfall instead of VaR
5. **Conservative calibration:**
   - Use 99.5% instead of 99%
   - Build in a safety buffer

Answer E.2: Lease Rate vs. Interest Rate

a) Why lease rates $<$ EUR rates:
Typical observation: $L \approx 0.2\%$–1.0%, EUR rates $\approx 2\%$–4%
Reasons:

1. **Gold pays no interest:**
   - No coupons like bonds
   - The lease rate is a "cost of holding"
   - Not a "return on investment"
2. **Storage costs:**
   - Gold incurs storage costs
   - Insurance, security
   - The lease rate can reflect these
3. **Convenience yield:**
   - Physical gold has value from "being available"
   - Industrial users pay a premium
   - Reduces the net lease rate
4. **Supply/demand:**
   - When many want to "lend" gold
   - and few want to "borrow"
   - $\rightarrow$ low lease rate

b) If lease $>$ EUR rates:
"Backwardation" in the gold market
Meaning:

- Borrowing gold is more expensive than borrowing EUR
- Could indicate scarcity
- Arbitrage opportunity:
  1. Borrow EUR at 3%
  2. Buy gold
  3. Lend the gold at 5%
  4. Risk-free profit of 2%

In practice:

- Would push lease rates down (more supply)
- Self-correcting mechanism
- Temporarily possible in crises

c) Historical examples:
September 1999:

- Lease rates rose to 9% (!)
- Reason: Central Bank Gold Agreement
- Fear of a gold shortage
- "Gold squeeze"

2008 financial crisis:

- Lease rates briefly above EUR rates
- Liquidity crunch
- Everyone wanted physical gold, nobody wanted to lend

2013 Cyprus crisis:

- Lease rates rose
- Concern about bank collapses
- Flight to physical gold

Answers E.3–E.5:

Detailed answers to E.3–E.5 would go beyond the scope of this collection. Key points:
E.3 (Practical limitations):

- 180–250 days optimal (trade-off: stability vs. timeliness)
- Weekly update recommended
- If lease rates are missing: implied derivation from futures, proxies, modelling

E.4 (Alternative methods):

- Parametric: fast, simple, but assumes a normal distribution
- Historical: no assumptions, but needs a lot of data
- Monte Carlo: flexible, but computationally intensive
- Best practice: use all three!

E.5 (Business sense):
Why gold swaps:

- Gold mines: price hedging of future production
- Jewellers/manufacturers: stable input costs
- Central banks: FX liquidity or a return on reserves
- Hedging vs. speculation: depends on the underlying exposure

Concrete examples:
Example 1: Barrick Gold (producer)

- Largest gold producer worldwide
- Uses gold swaps/forwards for price hedging
- Locks in future production at fixed prices
- Trade-off: stability vs. "selling the upside"

Example 2: Rolex/jewellery industry (manufacturer)

- Needs large quantities of gold for production
- Gold swaps to fix input costs
- Alternative: physical purchase, but storage costs and tied-up capital
- Predictable production costs

Example 3: Deutsche Bundesbank (central bank – reserves)

- Large gold reserves (3,350 tonnes, second-largest worldwide)
- Could theoretically earn a lease rate via swaps
- Let the gold "work" instead of storing it unused
- Politically sensitive, therefore rarely used

Example 4: Central Bank of Turkey (central bank – liquidity)

- **Situation 2020–2021:** low FX reserves, USD shortage
- **Solution:** gold swaps with commercial banks
- **Mechanics:** gold temporarily exchanged for USD/EUR
- **Advantage:** gold stays on the reserve balance sheet, cheaper than a loan
- **Risk:** gold price exposure, counterparty risk
- **Politics:** less sensitive than an IMF programme

Further historical examples:

- **Venezuela:** attempted gold swaps with Russia/China for USD
- **India (1991):** gold swaps during the balance-of-payments crisis
- **Egypt:** gold swaps to support FX reserves
- **Ghana, Uganda:** African central banks use gold for liquidity

Key insight:

- Gold swaps are versatile instruments

- Their use depends strongly on the institution and the situation
- For central banks: a critical tool for FX liquidity without selling gold
- For producers/consumers: a hedging instrument
- Political and accounting aspects matter

7 Final Remarks

7.1 For Students

This exercise collection is intended as supplementary material to the gold swap documentation. We recommend that you:

- Work through the exercises in the given order
- Use the model solutions to check your own work
- Experiment with the calculator for a deeper understanding
- Discuss the case studies in groups
- Ask questions whenever something is unclear

7.2 For Lecturers

Suggested use:

- **Part A:** After each lecture unit (2–3 exercises)
- **Part B:** Weekly homework
- **Part C:** Lab sessions with the calculator
- **Part D:** Group projects (2–3 students)
- **Part E:** Seminar discussions

Suggested assessments:

- **Written exam:** Parts A + B (90 min)
- **Practical exam:** Part C (45 min at the computer)
- **Essay:** Part D (take-home, 1 week)
- **Oral exam:** Part E (20 min per student)

7.3 Time Required

Part	Working time	Focus
Part A	2–3 hours	Concepts
Part B	4–5 hours	Calculation
Part C	3–4 hours	Practice
Part D	2–3 hours	Application
Part E	1–2 hours	Discussion
Total	**12–17 hours**	

Recommended: 1–2 weeks of working time

Good luck!
