Swap VaR Calculator

Risk Management with Historical Time Series

Mathematical Documentation

English

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1 Introduction

This document describes the mathematical foundations of two VaR calculators for interest rate swaps:

1. **Plain Vanilla Swap VaR Calculator:** swaps starting today.
2. **Forward Swap VaR Calculator:** forward starting swaps with lead time T_s. With $T_s = 0$ it is equivalent to the plain vanilla swap.

Both calculators use the same methodology:

- **valuation** with the current yield curve (last row of the time series),
- **risk factors** are the discount factors $\mathrm{DF}(t_i)$ at the payment dates,
- **covariance matrix** of the daily log returns of these discount factors from at least 251 trading days,
- **sensitivities** are the full derivatives of the present value with respect to $\ln \mathrm{DF}(t_i)$ (fixed *and* floating leg; for the forward swap also with respect to $\ln \mathrm{DF}(T_s)$),
- **VaR** using the variance-covariance (delta-normal) approach with a historically estimated covariance matrix.

2 Input Data

2.1 Interest Rate Time Series

The calculators read a time series of zero-coupon rates (continuously compounded, in percent):

$$r_k(T_j), \qquad k = 1, \ldots, K \text{ (days)}, \quad j = 1, \ldots, m \text{ (maturities)}, \tag{1}$$

with $K \geq 251$. The header row contains the maturities (`M` = months, `Y` = years), e.g.

```
Date,0.25Y,0.5Y,1Y,2Y,3Y,5Y,7Y,10Y
```

The **last row** provides the current yield curve $r(T_j) = r_K(T_j)$.

2.2 Built-in Sample Data

The Plain Vanilla Swap VaR Calculator contains two EUR data sets with 251 trading days each (14 Nov 2025 to 30 Oct 2026):

- **EUR swap curve** (zero-coupon, bootstrapped from swap rates),
- **EUR government bonds (Bund)** (zero-coupon, bootstrapped from Bund yields).

The data are market-like and synthetically generated. The Forward Swap VaR Calculator contains different sample series (maturities 1Y to 5Y); the EUR swap curve can be loaded there as a CSV file (Section 8). All numerical examples in this document use the EUR swap curve; the current curve as of 30 Oct 2026 is:

Maturity T_j	0.25	0.5	1	2	3	5	7	10
$r(T_j)$ in %	2.5916	2.7609	2.9267	3.0352	3.0246	3.0753	3.1377	3.2451

Table 1: EUR swap curve (zero-coupon, continuous) as of 30 Oct 2026

3 Plain Vanilla Swap

3.1 Definition and Parameters

In a plain vanilla interest rate swap one party pays a fixed rate and the other a floating rate on the same notional; only net payments are exchanged. The calculators value the swap from the perspective of the **fixed rate payer** (payer swap: receives floating, pays fixed).

Parameter	**Symbol**
Notional amount	N
Fixed interest rate (p.a.)	r_{fix}
Maturity (years)	T
Payment frequency (per year)	$f \in \{1, 2, 4\}$
Number of payments	$n = \lfloor T \cdot f \rfloor$
Payment interval (years)	$\Delta t = 1/f$

Table 2: Swap parameters

3.2 Payment Dates

$$t_i = i \cdot \Delta t, \qquad i = 1, \ldots, n. \tag{2}$$

Example (annual, 5 years): $t_i \in \{1, 2, 3, 4, 5\}$.

3.3 Yield Curve and Discount Factors

For **valuation**, the current curve is interpolated linearly between the grid points and extrapolated flat outside:

$$r(t) = \begin{cases} r(T_1), & t \le T_1, \\ r(T_j) + \dfrac{r(T_{j+1}) - r(T_j)}{T_{j+1} - T_j}\,(t - T_j), & T_j \le t \le T_{j+1}, \\ r(T_m), & t \ge T_m. \end{cases} \tag{3}$$

The discount factor is

$$\mathrm{DF}(t) = e^{-r(t)\cdot t}. \tag{4}$$

3.4 Forward Rates

The forward rate for the period $[t_{i-1}, t_i]$ follows from the discount factors:

$$F_i = \frac{1}{t_i - t_{i-1}} \left(\frac{\mathrm{DF}(t_{i-1})}{\mathrm{DF}(t_i)} - 1 \right), \qquad t_0 = 0. \tag{5}$$

For the first period the calculator uses $\mathrm{DF}(t_0)$ with $t_0 = 0.001$ years, i.e. $\mathrm{DF}(t_0) = e^{-r(T_1)\cdot 0.001} \approx 1$.

3.5 Cash Flows

$$C_{fix} = N \cdot r_{fix} \cdot \Delta t, \tag{6}$$
$$C_{float,i} = N \cdot F_i \cdot \Delta t, \tag{7}$$
$$C_{net,i} = C_{float,i} - C_{fix}. \tag{8}$$

3.6 Present Value

$$\boxed{\mathrm{PV} = \sum_{i=1}^{n} C_{net,i} \cdot \mathrm{DF}(t_i)} \tag{9}$$

A positive present value means the swap is in the money for the fixed rate payer.

4 Forward Starting Swap

4.1 Parameters and Payment Dates

A forward swap starts after the lead time T_s (forward start) and runs for T years (tenor):

$$t_i = T_s + i \cdot \Delta t, \qquad i = 1, \ldots, n, \qquad n = \lfloor T \cdot f \rfloor, \qquad T_e = T_s + T. \tag{10}$$

Example ($T_s = 2$, $T = 3$, annual): $t_i \in \{3, 4, 5\}$ – a "2Y-into-3Y" swap (2Yx3Y).

4.2 Forward Rates

$$F_1 = \frac{1}{t_1 - T_s}\left(\frac{\mathrm{DF}(T_s)}{\mathrm{DF}(t_1)} - 1\right), \tag{11}$$
$$F_i = \frac{1}{t_i - t_{i-1}}\left(\frac{\mathrm{DF}(t_{i-1})}{\mathrm{DF}(t_i)} - 1\right), \qquad i \geq 2. \tag{12}$$

Cash flows and present value as in Section 3.

4.3 Forward Swap Rate

The fair fixed rate (zero present value) is

$$r_{fwd} = \frac{\sum_{i=1}^{n} F_i \cdot \mathrm{DF}(t_i)}{\sum_{i=1}^{n} \mathrm{DF}(t_i)}. \tag{13}$$

For $T_s = 0$ this is the par swap rate.

5 Risk Metrics

All metrics are computed numerically by a parallel shift of the current curve by $\Delta r = 0.0001$ (1 bp); the forward rates are recalculated with the shifted curve.

5.1 DV01

$$\mathrm{DV01} = -\big(\mathrm{PV}(r + \Delta r) - \mathrm{PV}(r)\big). \tag{14}$$

A **negative** DV01 means that the present value **increases** when rates rise – typical for the fixed rate payer.

5.2 Modified Duration

Duration is expressed relative to the **notional** N:

$$D_{mod} = \frac{\mathrm{DV01} \cdot 10{,}000}{N} \approx -\frac{1}{N}\frac{\partial \mathrm{PV}}{\partial r}, \qquad \Delta\mathrm{PV} \approx -D_{mod} \cdot N \cdot \Delta r. \tag{15}$$

If all rates rise by one percentage point, the present value changes by about $-D_{mod}$ percent of the notional. For the fixed rate payer D_{mod} is negative.

Relating duration to the notional is common for swaps: a swap's present value is often close to zero (PV ≈ 0 at inception at market terms). A duration relative to |PV| would then become very large and could change sign; the notional-based duration remains stable.

5.3 Convexity

$$\mathrm{Convexity} = \frac{\mathrm{PV}(r + \Delta r) + \mathrm{PV}(r - \Delta r) - 2\,\mathrm{PV}(r)}{N \cdot (\Delta r)^2}. \tag{16}$$

Convexity is also relative to the notional; its sign is determined by the numerator alone. Together, the second-order approximation is $\Delta\mathrm{PV} \approx N\big(-D_{mod}\,\Delta r + \frac{1}{2}\,\mathrm{Convexity} \cdot (\Delta r)^2\big)$. For the fixed rate payer convexity is typically negative.

6 Value at Risk

6.1 Discount Factor Time Series

For each day k and each payment date t_i the day's yield curve is interpolated and the discount factor is formed:

$$\mathrm{DF}_{k,i} = e^{-r_k(t_i) \cdot t_i}. \tag{17}$$

Between grid points the interpolation is linear; beyond the longest maturity the curve is extended flat. For $t < T_1$ the discount factor is interpolated linearly between $\mathrm{DF}(0) = 1$ and $\mathrm{DF}(T_1)$:

$$\mathrm{DF}_k(t) = 1 + \big(\mathrm{DF}_k(T_1) - 1\big)\frac{t}{T_1}. \tag{18}$$

6.2 Log Returns

$$R_{k,i} = \ln \frac{\mathrm{DF}_{k,i}}{\mathrm{DF}_{k-1,i}} = -\big(r_k(t_i) - r_{k-1}(t_i)\big) \cdot t_i, \qquad k = 2, \ldots, K. \tag{19}$$

With $K = 251$ days there are 250 observations.

6.3 Covariance and Correlation Matrix

$$\bar{R}_i = \frac{1}{K-1}\sum_{k=2}^{K} R_{k,i}, \tag{20}$$

$$\Sigma_{ij} = \frac{1}{K-2}\sum_{k=2}^{K}(R_{k,i}-\bar{R}_i)(R_{k,j}-\bar{R}_j), \tag{21}$$

$$\sigma_i = \sqrt{\Sigma_{ii}}, \qquad \rho_{ij} = \frac{\Sigma_{ij}}{\sigma_i\,\sigma_j}. \tag{22}$$

$\mathbf{\Sigma}$ is symmetric, positive semi-definite and of dimension $n \times n$; for the forward swap the discount factor $\mathrm{DF}(T_s)$ is an additional risk factor ($(n+1)\times(n+1)$).

6.4 Sensitivity Vector

The floating cash flows themselves depend on the discount factors via the forward rates. With t_0 as the start of the first accrual period ($t_0 = T_s$ for the forward swap, $t_0 = 0.001$ years for the plain vanilla swap) the floating leg telescopes, and the present value is

$$\mathrm{PV} = N\,\mathrm{DF}(t_0) - N\,\mathrm{DF}(t_n) - N\,r_{fix}\,\Delta t\sum_{i=1}^{n}\mathrm{DF}(t_i). \tag{23}$$

Since the covariance matrix is estimated for **log returns**, the relevant sensitivities are the derivatives with respect to $\ln \mathrm{DF}(t_i)$:

$$\boxed{\begin{gathered} w_i = -N\,r_{fix}\,\Delta t\,\mathrm{DF}(t_i)\ \ (i<n), \qquad w_n = -N\,(1+r_{fix}\Delta t)\,\mathrm{DF}(t_n), \\ w_0 = N\,\mathrm{DF}(T_s) \end{gathered}} \tag{24}$$

The term w_0 is dropped for the plain vanilla swap, because $\mathrm{DF}(0.001) \approx 1$ practically does not move. For the forward swap, $\sum_{i=0}^{n} w_i = \mathrm{PV}$.

The VaR thus captures the fixed *and* the floating leg. Earlier versions used $w_i = C_{net,i}\mathrm{DF}(t_i)$, i.e. they froze the net cash flows; the VaR was therefore far too small (in the example below 40.81 instead of 4,817.78 EUR).

6.5 Portfolio Variance and VaR

$$\sigma_P^2 = \mathbf{w}^T\mathbf{\Sigma}\,\mathbf{w} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\,\Sigma_{ij}\,w_j, \tag{25}$$

$$\mathrm{VaR}_\alpha(h) = z_\alpha \cdot \sigma_P \cdot \sqrt{h}, \tag{26}$$

with $z_\alpha = \Phi^{-1}(\alpha)$ and holding period h in days: $z_{0.95} = 1.645$, $z_{0.99} = 2.326$, $z_{0.999} = 3.090$.

$$\boxed{\mathrm{VaR}_{99\%,1d} = 2.326 \cdot \sqrt{\mathbf{w}^T\mathbf{\Sigma}\,\mathbf{w}}} \tag{27}$$

6.6 Statistical Key Figures

The calculators also display:

$$\bar{\sigma} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\Sigma_{ii}} \qquad \text{(average daily volatility)}, \tag{28}$$

$$\bar{\rho} = \frac{1}{n-1}\sum_{i=1}^{n-1}\rho_{i,i+1} \qquad \text{(average correlation of adjacent dates)}. \tag{29}$$

For a single payment ($n = 1$) $\bar{\rho}$ is undefined; the calculator then shows "–".

7 Numerical Example: Plain Vanilla Swap

7.1 Parameters

The values correspond to the calculator's default settings with the sample data "EUR swap curve".

Parameter	**Value**
Notional N	1,000,000 EUR
Fixed rate r_{fix}	3.5% p.a.
Maturity T	5 years
Payment frequency f	annual
Number of payments n	5
Confidence level / holding period	99% / 1 day

7.2 Discount Factors, Forward Rates and Cash Flows

The rates for $t = 1, 2, 3, 5$ are grid points; $r(4) = (3.0246 + 3.0753)/2 = 3.0500\%$. The fixed payment is $C_{fix} = 1{,}000{,}000 \cdot 0.035 = 35{,}000$ EUR.

t_i	$r(t_i)$	$\mathrm{DF}(t_i)$	F_i	$C_{float,i}$	$C_{net,i}$	$C_{net,i}\mathrm{DF}(t_i)$
1	2.9267%	0.971157	2.9673%	29,672.80	−5,327.20	−5,173.55
2	3.0352%	0.941102	3.1936%	31,936.36	−3,063.64	−2,883.20
3	3.0246%	0.913257	3.0490%	30,489.57	−4,510.43	−4,119.18
4	3.0500%	0.885150	3.1754%	31,753.72	−3,246.28	−2,873.44
5	3.0753%	0.857474	3.2277%	32,276.96	−2,723.04	−2,334.94
Sum = present value						**−17,384.30**

Table 3: Cash flows in EUR (swap curve as of 30 Oct 2026)

Example first period:

$$F_1 = \frac{1}{1}\left(\frac{0.999974}{0.971157} - 1\right) = 0.029673 = 2.9673\%. \tag{30}$$

The present value is negative because the fixed rate (3.5%) is above the par swap rate of 3.12%.

7.3 Risk Metrics

$$\mathrm{PV}(r+1\,\mathrm{bp}) = -16{,}908.81 \text{ EUR}, \qquad \mathrm{PV}(r-1\,\mathrm{bp}) = -17{,}860.03 \text{ EUR}, \tag{31}$$

$$\mathrm{DV01} = -\big(-16{,}908.81 - (-17{,}384.30)\big) = -475.49 \text{ EUR}, \tag{32}$$

$$D_{mod} = -475.49 \cdot 10{,}000/1{,}000{,}000 = -4.75, \tag{33}$$

$$\text{Convexity} = \frac{-16{,}908.81 - 17{,}860.03 + 2 \cdot 17{,}384.30}{1{,}000{,}000 \cdot 10^{-8}} = -23.14. \tag{34}$$

The exact numerator of the convexity is -0.2314 EUR; the rounded present values give -0.24. For a rise in rates of 1 percentage point the duration implies a gain of about 4.75 % of the notional, i.e. roughly 47,500 EUR.

7.4 Discount Factor Time Series and Returns

Day	DF(1)	DF(2)	DF(3)	DF(4)	DF(5)
1 (14 Nov 2025)	0.980362	0.959378	0.938355	0.915488	0.891634
2 (17 Nov 2025)	0.980351	0.959497	0.937947	0.914604	0.890128
3 (18 Nov 2025)	0.980151	0.958728	0.937500	0.914116	0.889647
⋮	⋮	⋮	⋮	⋮	⋮
251 (30 Oct 2026)	0.971157	0.941102	0.913257	0.885150	0.857474

Table 4: Discount factors at the payment dates (extract)

Log returns on day 2, e.g. $R_{2,1} = -(1.9845 - 1.9833)\% \cdot 1 = -0.000012$:

$$\mathbf{R}_2 = (-1.20;\ 12.40;\ -43.50;\ -96.60;\ -169.00) \times 10^{-5}. \tag{35}$$

7.5 Covariance and Correlation Matrix

From 250 observations:

$$\boldsymbol{\Sigma} = \begin{pmatrix} 6.158 & 16.129 & 25.377 & 33.251 & 40.832 \\ 16.129 & 52.634 & 78.508 & 105.915 & 133.941 \\ 25.377 & 78.508 & 144.405 & 191.640 & 238.427 \\ 33.251 & 105.915 & 191.640 & 269.389 & 354.071 \\ 40.832 & 133.941 & 238.427 & 354.071 & 487.799 \end{pmatrix} \times 10^{-8} \tag{36}$$

Daily volatilities σ_i: 0.0248%, 0.0725%, 0.1202%, 0.1641%, 0.2209% – they increase with maturity because $R_{k,i} = -\Delta r \cdot t_i$.

$$\boldsymbol{\rho} = \begin{pmatrix} 1.000 & 0.896 & 0.851 & 0.816 & 0.745 \\ 0.896 & 1.000 & 0.901 & 0.890 & 0.836 \\ 0.851 & 0.901 & 1.000 & 0.972 & 0.898 \\ 0.816 & 0.890 & 0.972 & 1.000 & 0.977 \\ 0.745 & 0.836 & 0.898 & 0.977 & 1.000 \end{pmatrix} \tag{37}$$

7.6 Portfolio Variance and VaR

Sensitivity vector with $N\, r_{fix}\Delta t = 35{,}000$ EUR:

$$w_i = -35{,}000 \cdot \mathrm{DF}(t_i) \;\; (i = 1, \ldots, 4), \qquad w_5 = -1{,}035{,}000 \cdot \mathrm{DF}(5),$$
$$\mathbf{w} = (-33{,}990.50;\; -32{,}938.56;\; -31{,}963.99;\; -30{,}980.25;\; -887{,}485.08)^T \text{ EUR}. \tag{38}$$

Intermediate step $\mathbf{w}^T\boldsymbol{\Sigma}$, e.g. the first component:

$$[\mathbf{w}^T\boldsymbol{\Sigma}]_1 = \big(-33{,}990.50 \cdot 6.158 - 32{,}938.56 \cdot 16.129 - 31{,}963.99 \cdot 25.377 - 30{,}980.25 \cdot 33.251 - 887{,}485.08 \cdot 40.832\big) \times 10^{-8} = -0.3882, \tag{39}$$

in total $\mathbf{w}^T\boldsymbol{\Sigma} = (-0.3882;\; -1.2694;\; -2.2560;\; -3.3332;\; -4.5730)$.

$$\sigma_P^2 = \mathbf{w}^T\boldsymbol{\Sigma}\,\mathbf{w} = 4{,}288{,}894.64 \text{ EUR}^2, \qquad \sigma_P = 2{,}070.96 \text{ EUR}, \tag{40}$$
$$\mathrm{VaR}_{99\%,1d} = 2.3263 \cdot 2{,}070.96 = \mathbf{4{,}817.78} \text{ EUR}. \tag{41}$$

The largest contribution comes from the last payment (w_5), which carries the repayment effect of the floating leg in addition to the fixed rate.

Statistical key figures: $\bar{\sigma} = 0.1386\%$, $\bar{\rho} = (0.896 + 0.901 + 0.972 + 0.977)/4 = 0.9362$.

7.7 Semi-Annual Payments

With $f = 2$ (10 payments, $t_i = 0.5, 1.0, \ldots, 5.0$): PV $= -18{,}642.20$ EUR, DV01 $= -471.81$ EUR, $D_{mod} = -4.72$, convexity $= -22.93$, $\mathrm{VaR}_{99\%,1d} = 4{,}775.83$ EUR.

8 Numerical Example: Forward Swap

8.1 Parameters

Default settings of the Forward Swap VaR Calculator: $N = 1{,}000{,}000$ EUR, $r_{fix} = 3.5\%$, forward start $T_s = 2$, tenor $T = 3$, annual ($n = 3$), data: EUR swap curve.

The EUR swap curve is not built into the Forward Swap VaR Calculator. To reproduce the example, save it in the Plain Vanilla Swap VaR Calculator with “As CSV” and load it in the Forward Swap VaR Calculator as a time series file.

8.2 Cash Flows

With $\mathrm{DF}(2) = 0.941102$:

$$F_1 = \frac{1}{3-2}\left(\frac{0.941102}{0.913257} - 1\right) = 0.030490 = 3.0490\%. \tag{42}$$

t_i	DF(t_i)	F_i	$C_{net,i}$	w_i
2	0.941102	–	–	941,101.76
3	0.913257	3.0490%	−4,510.43	−31,963.99
4	0.885150	3.1754%	−3,246.28	−30,980.25
5	0.857474	3.2277%	−2,723.04	−887,485.08
Present value $= \sum_i w_i$				**−9,327.56**

Table 5: Forward swap 2Yx3Y (EUR)

The forward swap rate is 3.15%.

8.3 Risk

DV01 = −277.41 EUR, $D_{mod} = -2.77$, convexity = −19.21;

Risk factors DF(2), . . . , DF(5); σ_P = 1,516.46 EUR, $\text{VaR}_{99\%,1d}$ = 3,527.82 EUR, $\bar{\sigma} = 0.1734\%$, $\bar{\rho} = 0.9742$.

8.4 Comparison

Key figure	**5Y annual**	**5Y semi-annual**	**Forward 2Yx3Y**
Payments	5	10	3
Present value (EUR)	−17,384.30	−18,642.20	−9,327.56
DV01 (EUR)	−475.49	−471.81	−277.41
Modified duration	−4.75	−4.72	−2.77
VaR 99%, 1 day (EUR)	4,817.78	4,775.83	3,527.82

Table 6: Comparison (EUR swap curve, 30 Oct 2026)

The forward swap has fewer payments and a lower interest rate risk; its VaR is not proportionally smaller, because the remaining payments lie on the more volatile long maturities.

9 Implementation Details

- **Log returns** instead of arithmetic returns: time-additive and symmetric.
- **Bessel's correction**: division by $K - 2$ (number of returns minus 1).
- **Validation** (console): symmetry $|\Sigma_{ij} - \Sigma_{ji}| < 10^{-15}$, correlations in $[-1, 1]$, check for identical columns.
- **Import**: comma, semicolon or tab as delimiter; with semicolon a decimal comma is accepted; columns are sorted by maturity; incomplete rows are skipped.
- **Minimum length**: 251 complete data rows.

10 References

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11 Appendix: Formula Summary

Discount factor	$\mathrm{DF}(t) = e^{-r(t)\,t}$
Forward rate	$F_i = \frac{1}{t_i - t_{i-1}}\big(\mathrm{DF}(t_{i-1})/\mathrm{DF}(t_i) - 1\big)$
Net cash flow	$C_{net,i} = N\,(F_i - r_{fix})\,\Delta t$
Present value	$\mathrm{PV} = \sum_i C_{net,i}\,\mathrm{DF}(t_i)$
DV01	$\mathrm{DV01} = -\big(\mathrm{PV}(r + \Delta r) - \mathrm{PV}(r)\big)$
Modified duration	$D_{mod} = \mathrm{DV01} \cdot 10{,}000/N$
Convexity	$\big(\mathrm{PV}(r+\Delta r) + \mathrm{PV}(r-\Delta r) - 2\mathrm{PV}(r)\big)/\big(N(\Delta r)^2\big)$
Log return	$R_{k,i} = \ln(\mathrm{DF}_{k,i}/\mathrm{DF}_{k-1,i})$
Covariance	$\Sigma_{ij} = \frac{1}{K-2}\sum_k (R_{k,i} - \bar{R}_i)(R_{k,j} - \bar{R}_j)$
Sensitivity	$w_i = -N r_{fix} \Delta t\,\mathrm{DF}(t_i)\ (i < n)$, $w_0 = N\,\mathrm{DF}(T_s)$
	$w_n = -N(1 + r_{fix}\Delta t)\,\mathrm{DF}(t_n)$
Value at Risk	$\mathrm{VaR}_\alpha = z_\alpha \sqrt{\mathbf{w}^T \boldsymbol{\Sigma}\, \mathbf{w}}\,\sqrt{h}$

Symbol	Meaning
N	Notional amount
r_{fix}	Fixed interest rate
T, T_s	Maturity (tenor), forward start
f, Δt	Payment frequency, payment interval
n	Number of payments
t_i	Payment date i
T_j, m	Grid points of the yield curve, their number
K	Number of days in the time series
$r(t)$	Zero rate (continuous) for maturity t
$\mathrm{DF}(t)$	Discount factor
F_i	Forward rate of period i
C_{fix}, $C_{float,i}$, $C_{net,i}$	Fixed, floating, net payment
$R_{k,i}$	Log return of $\mathrm{DF}(t_i)$ on day k
$\boldsymbol{\Sigma}$, $\boldsymbol{\rho}$	Covariance, correlation matrix
$\mathbf{w}$	Sensitivity vector
σ_P	Portfolio standard deviation
z_α, h	Normal quantile, holding period in days