Bond Valuation and Risk Calculator

Mathematical Calculation Steps

English

Wolfgang Boehme

wolfgang.boehme@economic-modeling.de

www.economic-modeling.de

25 October 2026

The documentation on this website was developed with the support of the AI Claude (Anthropic).

Table of Contents

1 Fundamental Concepts **2**
1.1 Yield Curve and Discounting . . . 2
1.2 Yield Curve Interpolation . . . 2

2 Fixed Rate Bonds **2**
2.1 Bond Parameters . . . 2
2.2 Coupon Payments . . . 2
2.3 Present Value . . . 3
2.4 Clean Price and Dirty Price . . . 3
2.5 Macaulay Duration . . . 3
2.6 Modified Duration . . . 3
2.7 DV01 (Dollar Value of 01) . . . 4
2.8 Convexity . . . 4

3 Floating Rate Bonds **4**
3.1 Bond Parameters . . . 4
3.2 Forward rates . . . 4
3.3 Variable Coupon Payments . . . 5
3.4 Present Value . . . 5
3.5 Accrued Interest . . . 5

4 Zero-Coupon Bonds **5**
4.1 Bond Parameters . . . 5
4.2 Present Value . . . 5
4.3 Clean and Dirty Price . . . 5
4.4 Macaulay Duration . . . 6
4.5 Modified Duration . . . 6
4.6 DV01 und Convexity . . . 6

5 Value at Risk (VaR) **6**
5.1 Two Methods . . . 6
5.2 Parametric VaR . . . 6
5.3 Empirical Portfolio VaR with Historical Data . . . 7
5.3.1 Historical Data Structure . . . 7
5.3.2 Calculating Returns . . . 7
5.3.3 Empirical Variance-Covariance Matrix . . . 7
5.3.4 Volatilities . . . 8
5.3.5 Correlations . . . 8
5.3.6 DV01 Vector . . . 8
5.3.7 Portfolio Variance . . . 8
5.3.8 Scaling to Time Horizon . . . 9
5.3.9 Portfolio VaR . . . 9
5.4 Z-Score Calculation . . . 9

6 Statistical Analyses **9**
6.1 Cash Flow Statistics 9
6.2 Covariance Between Consecutive Cash Flows 10
6.3 Correlation 10

7 Scenario Analysis **10**

8 Yield to Maturity (YTM) **10**

9 Implementation Notes **10**
9.1 Numerical Stability 10
9.2 Yield Curve Management 11
9.3 Trading Day Convention 11

10 Worked Example 1: Fixed Rate Bond **11**
10.1 Given 11
10.2 Calculations 11

11 Worked Example 2: Floating Rate Bond **12**
11.1 Given 12
11.2 Calculations 12

12 Worked Example 3: Portfolio VaR with Historical Data **13**
12.1 Given 13
12.2 Empirical Covariance Matrix (Example Extract) 13
12.3 DV01 Vector 14
12.4 Portfolio Variance 14
12.5 Scaling to 10 Days 14
12.6 VaR 14

13 Formula Summary **15**

14 References **15**

1 Fundamental Concepts

1.1 Yield Curve and Discounting

The yield curve $r(t)$ gives the interest rate for maturity t. For valuation we use continuous discounting:

$$DF(t) = e^{-r(t) \cdot t} \tag{1}$$

where:

- $DF(t)$ = discount factor for maturity t
- $r(t)$ = interest rate from the yield curve for maturity t
- t = time in years

1.2 Yield Curve Interpolation

For maturities between nodes we use linear interpolation:

$$r(t) = r_1 + \frac{r_2 - r_1}{t_2 - t_1} \cdot (t - t_1) \tag{2}$$

where $t_1 \leq t \leq t_2$ and r_1, r_2 are the interest rates at t_1, t_2.

2 Fixed Rate Bonds

2.1 Bond Parameters

- N = nominal amount
- c = coupon (annual rate)
- T = term in years
- f = payment frequency (1=annual, 2=semi-annual, 4=quarterly, 12=monthly)
- n = number of payments = $\lfloor T \cdot f \rfloor$
- Δt = payment interval = $\frac{1}{f}$

2.2 Coupon Payments

The coupon payment per period is:

$$C = \frac{N \cdot c}{f} \tag{3}$$

The payment dates are:

$$t_i = i \cdot \Delta t \quad \text{für } i = 1, 2, \ldots, n \tag{4}$$

2.3 Present Value

The present value is the sum of all discounted coupon payments plus the discounted redemption:

$$PV = \sum_{i=1}^{n} C \cdot DF(t_i) + N \cdot DF(t_n) \tag{5}$$

Written out:

$$PV = \sum_{i=1}^{n} C \cdot e^{-r(t_i) \cdot t_i} + N \cdot e^{-r(t_n) \cdot t_n} \tag{6}$$

2.4 Clean Price and Dirty Price

Clean Price (excluding accrued interest):

$$\text{Clean Price} = \frac{PV}{N} \cdot 100\% \tag{7}$$

Accrued Interest:

$$AI = \frac{C \cdot d}{d_{\text{Periode}}} \tag{8}$$

where:

- d = days since last coupon payment
- d_{Periode} = days in one period = $\frac{365}{f}$

Dirty Price (including accrued interest):

$$\text{Dirty Price} = \text{Clean Price} + \frac{AI}{N} \cdot 100\% \tag{9}$$

2.5 Macaulay Duration

The Macaulay Duration measures the average capital commitment period:

$$D_{\text{Mac}} = \frac{1}{PV} \left[\sum_{i=1}^{n} t_i \cdot C \cdot DF(t_i) + t_n \cdot N \cdot DF(t_n) \right] \tag{10}$$

2.6 Modified Duration

The Modified Duration measures the percentage price change per unit rate change:

$$D_{\text{Mod}} = \frac{D_{\text{Mac}}}{1 + \frac{\bar{r}}{f}} \tag{11}$$

where $\bar{r}$ is the average interest rate (e.g. $r(T/2)$).

2.7 DV01 (Dollar Value of 01)

DV01 measures the absolute value change for 1 basis point (0.01%) rate change:

$$DV01 = -\left[PV(r + 0.0001) - PV(r)\right] \tag{12}$$

Berechnung:

1. Increase all rates in the curve by 1bp: $r'(t) = r(t) + 0.0001$
2. Calculate PV' with the shifted curve
3. $DV01 = -(PV' - PV)$

2.8 Convexity

Convexity measures the curvature of the price-yield relationship:

$$\text{Convexity} = \frac{PV^+ + PV^- - 2 \cdot PV}{PV \cdot (\Delta r)^2} \tag{13}$$

where:

- PV^+ = Present Value at $r(t) + \Delta r$
- PV^- = Present Value at $r(t) - \Delta r$
- $\Delta r = 0.0001$ (1 basis point)

3 Floating Rate Bonds

3.1 Bond Parameters

- N = nominal amount
- s = spread over reference rate (in basis points)
- T = term in years
- f = Zahlungsfrequenz
- n = number of payments = $\lfloor T \cdot f \rfloor$

3.2 Forward rates

For each payment date we calculate the forward rate from the yield curve:

First period:

$$f_1 = r(t_1) \tag{14}$$

Subsequent periods:

$$f_i = \frac{DF(t_{i-1})/DF(t_i) - 1}{t_i - t_{i-1}} \tag{15}$$

Written out:

$$f_i = \frac{e^{-r(t_{i-1}) \cdot t_{i-1}} / e^{-r(t_i) \cdot t_i} - 1}{t_i - t_{i-1}} \tag{16}$$

3.3 Variable Coupon Payments

The coupon payment for period i is:

$$C_i = \frac{N \cdot (f_i + s)}{f_{\text{freq}}} \tag{17}$$

where:

- f_i = forward rate for period i
- s = spread (as decimal, e.g. 150bp = 0.015)
- f_{freq} = payment frequency

3.4 Present Value

$$PV = \sum_{i=1}^{n} C_i \cdot DF(t_i) + N \cdot DF(t_n) \tag{18}$$

3.5 Accrued Interest

Estimated using the first forward rate:

$$AI = \frac{C_1 \cdot d}{d_{\text{Periode}}} \tag{19}$$

wobei $C_1 = \frac{N \cdot (f_1 + s)}{f_{\text{freq}}}$

4 Zero-Coupon Bonds

4.1 Bond Parameters

- N = nominal amount / Rückzahlung
- T = remaining term in years

4.2 Present Value

For zero-coupon bonds there is only one payment at maturity:

$$PV = N \cdot DF(T) = N \cdot e^{-r(T) \cdot T} \tag{20}$$

4.3 Clean and Dirty Price

Since there are no coupon payments:

$$\text{Clean Price} = \text{Dirty Price} = \frac{PV}{N} \cdot 100\% \tag{21}$$

$$\text{Accrued Interest} = 0 \tag{22}$$

4.4 Macaulay Duration

For zero-coupon bonds the duration equals the term:

$$D_{\text{Mac}} = T \tag{23}$$

4.5 Modified Duration

$$D_{\text{Mod}} = \frac{T}{1 + r(T)} \tag{24}$$

4.6 DV01 und Convexity

Analogous to fixed rate bonds, but with a single payment:

$$DV01 = -\left[N \cdot e^{-(r(T)+0.0001)\cdot T} - N \cdot e^{-r(T)\cdot T}\right] \tag{25}$$

$$\text{Convexity} = \frac{PV^{+} + PV^{-} - 2 \cdot PV}{PV \cdot (0.0001)^2} \tag{26}$$

5 Value at Risk (VaR)

5.1 Two Methods

The calculator supports two VaR methods:

1. **Parametric VaR** – with manual yield curve input
2. **Empirical Portfolio VaR** – with historical interest rate data

5.2 Parametric VaR

With manual yield curve input we use:

$$VaR = z_{\alpha} \cdot \sigma_{\text{scaled}} \cdot 10000 \cdot |DV01| \tag{27}$$

where:

- z_{α} = Z-score for confidence level α (e.g. 1.645 for 95%)
- σ_{scaled} = scaled volatility
- $DV01$ = Dollar Value of 01 (in EUR)

Volatility scaling:

$$\sigma_{\text{scaled}} = \sigma_{\text{täglich}} \cdot \sqrt{\frac{h}{250}} \tag{28}$$

where:

- σ_{daily} = daily interest rate volatility (in decimal form)
- h = time horizon in days
- 250 = number of trading days per year

5.3 Empirical Portfolio VaR with Historical Data

Methodology Overview:

- **Risk factors:** Zero rates (interest rates for various maturities)
- **Covariancematrix:** Aus historischen Zinsänderungen $\Delta r_{i,t}$
- **Sensitivities:** Key rate DV01 (numerical derivatives w.r.t. zero rates)
- **Consistency:** DV01 via numerical differentiation $\approx$ analytical formula $CF \cdot t \cdot DF$

5.3.1 Historical Data Structure

The CSV file contains:

- Date
- Interest rates for various maturities (e.g. 3M, 6M, 1Y, 2Y, 3Y, 5Y, 7Y, 10Y)
- At least 250 observations

5.3.2 Calculating Returns

The daily interest rate changes (returns) are calculated as:

$$\Delta r_{i,t} = r_{i,t} - r_{i,t-1} \tag{29}$$

where:

- i = maturity index (1, 2, ..., m)
- t = time index (day)
- $r_{i,t}$ = interest rate for maturity i on day t

5.3.3 Empirical Variance-Covariance Matrix

The empirical covariance between maturities i and j is calculated as:

$$\Sigma_{ij} = \frac{1}{T-1} \sum_{t=1}^{T} (\Delta r_{i,t} - \bar{\Delta r_i})(\Delta r_{j,t} - \bar{\Delta r_j}) \tag{30}$$

where:

- T = number of observations (returns)
- $\bar{\Delta r_i}$ = mean return for maturity i

Means:

$$\bar{\Delta r_i} = \frac{1}{T} \sum_{t=1}^{T} \Delta r_{i,t} \tag{31}$$

The full covariance matrix is:

$$\mathbf{\Sigma} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} & \cdots & \Sigma_{1m} \\ \Sigma_{21} & \Sigma_{22} & \cdots & \Sigma_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m1} & \Sigma_{m2} & \cdots & \Sigma_{mm} \end{pmatrix} \tag{32}$$

5.3.4 Volatilities

The volatility for maturity i is:

$$\sigma_i = \sqrt{\Sigma_{ii}} \tag{33}$$

5.3.5 Correlations

The correlation between maturities i and j is:

$$\rho_{ij} = \frac{\Sigma_{ij}}{\sigma_i \cdot \sigma_j} \tag{34}$$

The correlation matrix is:

$$\boldsymbol{\rho} = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1m} \\ \rho_{21} & 1 & \cdots & \rho_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{m1} & \rho_{m2} & \cdots & 1 \end{pmatrix} \tag{35}$$

5.3.6 DV01 Vector

For each maturity i in the yield curve we calculate the key rate DV01:

$$DV01_i = -\left[PV(r_i + 0.0001) - PV(r)\right] \tag{36}$$

where only the rate for maturity i is increased by 1bp; all others remain constant.
Analytical equivalence: This numerical derivative corresponds to the sensitivity:

$$DV01_i \approx -\frac{\partial PV}{\partial r_i} = \sum_{j:t_j \text{ nutzt } r_i} CF_j \cdot t_j \cdot DF_j \tag{37}$$

where the sum runs over all cash flows that depend on the zero rate r_i.
The DV01 vector is:

$$\mathbf{w} = \begin{pmatrix} DV01_1 \\ DV01_2 \\ \vdots \\ DV01_m \end{pmatrix} \tag{38}$$

5.3.7 Portfolio Variance

The portfolio variance (daily basis) is calculated as:

$$\sigma_P^2 = \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} = \sum_{i=1}^{m} \sum_{j=1}^{m} DV01_i \cdot \Sigma_{ij} \cdot DV01_j \tag{39}$$

The portfolio standard deviation is:

$$\sigma_P = \sqrt{\sigma_P^2} \tag{40}$$

5.3.8 Scaling to Time Horizon

The portfolio volatility is scaled to the time horizon:

$$\sigma_{P,h} = \sigma_P \cdot \sqrt{\frac{h}{250}} \tag{41}$$

where h is the time horizon in days.

5.3.9 Portfolio VaR

The final VaR is calculated as:

$$VaR = z_\alpha \cdot \sigma_{P,h} \cdot 10000 \tag{42}$$

The factor 10000 converts from decimal to basis points and then to EUR via the DV01 weighting.

5.4 Z-Score Calculation

The Z-score is calculated via the inverse normal distribution:

$$z_\alpha = \Phi^{-1}(\alpha) \tag{43}$$

Example values:

- 90% confidence: $z_{0.90} = 1.282$
- 95% confidence: $z_{0.95} = 1.645$
- 99% confidence: $z_{0.99} = 2.326$

6 Statistical Analyses

6.1 Cash Flow Statistics

Let the cash flows be $C_1, C_2, \ldots, C_n$.

Mean:

$$\mu = \frac{1}{n} \sum_{i=1}^{n} C_i \tag{44}$$

Variance:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (C_i - \mu)^2 \tag{45}$$

Standard deviation:

$$\sigma = \sqrt{\sigma^2} \tag{46}$$

6.2 Covariance Between Consecutive Cash Flows

$$\text{Cov}(C_i, C_{i+1}) = (C_i - \mu)(C_{i+1} - \mu) \tag{47}$$

Average covariance:

$$\bar{\text{Cov}} = \frac{1}{n-1} \sum_{i=1}^{n-1} (C_i - \mu)(C_{i+1} - \mu) \tag{48}$$

6.3 Correlation

$$\rho = \frac{\bar{\text{Cov}}}{\sigma^2} \tag{49}$$

7 Scenario Analysis

For scenario analysis we shift the entire yield curve in parallel:

$$r'(t) = r(t) + \Delta r \tag{50}$$

where $\Delta r \in \{-50, -25, -10, 0, +10, +25, +50\}$ basis points.
For each scenario we calculate:

- New present value: PV'
- Absolute change: $\Delta PV = PV' - PV$
- Percentage change: $\frac{\Delta PV}{|PV|} \cdot 100\%$
- New clean price: $\frac{PV'}{N} \cdot 100\%$

8 Yield to Maturity (YTM)

The yield to maturity is the internal rate of return that equates the present value with the market price.

For the approximation we use:

$$YTM \approx \bar{r} \tag{51}$$

where $\bar{r}$ is the average yield curve rate (e.g. at $T/2$).

Note: An exact YTM calculation requires iterative methods (e.g. Newton-Raphson), which are computationally expensive for complex yield curve structures.

9 Implementation Notes

9.1 Numerical Stability

- All interest rates are stored internally as decimals (e.g. 3.5% = 0.035)
- Basis points are converted by dividing by 10000
- Exponential functions use $e^{-r \cdot t}$ for continuous discounting

9.2 Yield Curve Management

- Linear interpolation between nodes
- Constant extrapolation outside the defined range
- Spread adjustments are added after interpolation

9.3 Trading Day Convention

For VaR calculations we use the convention:

- 250 trading days per year
- 365 calendar days for accrued interest calculations

10 Worked Example 1: Fixed Rate Bond

10.1 Given

- Nominal amount: $N = 1\,000\,000$ EUR
- Coupon: $c = 3.5\%$ p.a.
- Term: $T = 5$ years
- Frequency: $f = 2$ (semi-annual)
- Interest rate (constant): $r = 3.0\%$

10.2 Calculations

Number of payments:

$$n = \lfloor 5 \cdot 2 \rfloor = 10 \tag{52}$$

Coupon payment:

$$C = \frac{1,000,000 \cdot 0.035}{2} = 17,500 \text{ EUR} \tag{53}$$

Payment dates:

$$t_i = \{0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0\} \text{ years} \tag{54}$$

Present Value (first 3 payments + principal):

$$\begin{aligned} PV = {} & 17,500 \cdot e^{-0.03 \cdot 0.5} + 17,500 \cdot e^{-0.03 \cdot 1.0} + \ldots \\ & + 17,500 \cdot e^{-0.03 \cdot 5.0} + 1,000,000 \cdot e^{-0.03 \cdot 5.0} \end{aligned} \tag{55}$$

Numerical result:

$$PV \approx 1,023,186 \text{ EUR} \tag{56}$$

Clean Price:

$$\text{Clean Price} = \frac{1,023,186}{1,000,000} \cdot 100\% = 102.32\% \tag{57}$$

11 Worked Example 2: Floating Rate Bond

11.1 Given

- Nominal amount: $N = 1\,000\,000$ EUR
- Spread: $s = 150$ bp $= 0.015$
- Term: $T = 5$ years
- Frequency: $f = 4$ (quarterly)
- Yield curve: rising from 2.5% (3M) to 3.9% (10Y)

11.2 Calculations

Number of payments:

$$n = \lfloor 5 \cdot 4 \rfloor = 20 \tag{58}$$

Payment interval:

$$\Delta t = \frac{1}{4} = 0.25 \text{ years} \tag{59}$$

Forward rate Calculation (Example for Period 2):
Given:

- $t_1 = 0.25$ years, $r(t_1) = 2.5\%$
- $t_2 = 0.50$ years, $r(t_2) = 2.6\%$

$$\begin{aligned} f_2 &= \frac{DF(t_1)/DF(t_2) - 1}{t_2 - t_1} \\ &= \frac{e^{-0.025 \cdot 0.25}/e^{-0.026 \cdot 0.50} - 1}{0.50 - 0.25} \\ &= \frac{e^{-0.00625}/e^{-0.013} - 1}{0.25} \\ &= \frac{0.993769/0.987080 - 1}{0.25} \\ &= \frac{1.006778 - 1}{0.25} = 0.027112 = 2.71\% \end{aligned} \tag{60}$$

Coupon Payment Period 2:

$$C_2 = \frac{1,000,000 \cdot (0.02711 + 0.015)}{4} = 10,528 \text{ EUR} \tag{61}$$

Present Value of Period 2:

$$PV_2 = 10,528 \cdot e^{-0.026 \cdot 0.50} = 10,528 \cdot 0.987080 = 10,392 \text{ EUR} \tag{62}$$

Total Present Value:

For all 20 periods plus principal:

$$PV = \sum_{i=1}^{20} C_i \cdot DF(t_i) + 1,000,000 \cdot DF(5.0) \tag{63}$$

Numerical result (approximate):

$$PV \approx 1,045,230 \text{ EUR} \tag{64}$$

Clean Price:

$$\text{Clean Price} = \frac{1,045,230}{1,000,000} \cdot 100\% = 104.52\% \tag{65}$$

Interpretation:
The floating rate bond trades above par (100%) because:

- The additional spread of 150bp is attractive
- Forward rates are lower than the effective rate (forward + spread)
- The spread increases the value of all future payments

12 Worked Example 3: Portfolio VaR with Historical Data

12.1 Given

- Fixed rate bond: $N = 1\,000\,000$ EUR, $c = 3.5\%$, $T = 5$ years
- 252 historical observations (1 year)
- 8 maturities: 3M, 6M, 1Y, 2Y, 3Y, 5Y, 7Y, 10Y
- Confidence level: 95%
- Time horizon: 10 days

12.2 Empirical Covariance Matrix (Example Extract)

Calculated from historical data (in 10^{-6}):

$$\boldsymbol{\Sigma} = \begin{pmatrix} 9.5 & 8.1 & 6.7 & \cdots \\ 8.1 & 7.3 & 6.0 & \cdots \\ 6.7 & 6.0 & 4.8 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{66}$$

Volatilities (bp/day):

$$\boldsymbol{\sigma} = \begin{pmatrix} 3.08 \\ 2.70 \\ 2.19 \\ 1.98 \\ 1.85 \\ 1.71 \\ 1.60 \\ 1.50 \end{pmatrix} \tag{67}$$

12.3 DV01 Vector

For the 5-year bond with semi-annual payments:

$$\mathbf{w} = \begin{pmatrix} 158 \\ 312 \\ 615 \\ 1,205 \\ 1,765 \\ 2,850 \\ 1,920 \\ 850 \end{pmatrix} \text{ EUR} \tag{68}$$

12.4 Portfolio Variance

$$\sigma_P^2 = \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} = 0.000164 \tag{69}$$

$$\sigma_P = \sqrt{0.000164} = 0.0128 = 128 \text{ bp/Tag (in PV-Begriffen)} \tag{70}$$

12.5 Scaling to 10 Days

$$\sigma_{P,10} = 0.0128 \cdot \sqrt{\frac{10}{250}} = 0.0128 \cdot 0.2 = 0.00256 \tag{71}$$

12.6 VaR

$$VaR = 1.645 \cdot 0.00256 \cdot 10000 = 42.11 \text{ EUR} \tag{72}$$

Interpretation:
With 95% probability the loss over 10 days will not exceed EUR 42.11.

13 Formula Summary

Tabelle 1: Summary of Main Formulae

Metric	**Formula**	**Unit**
Discount factor	$e^{-r(t)\cdot t}$	-
Present Value	$\sum C_i \cdot DF(t_i) + N \cdot DF(t_n)$	EUR
Forward rate	$\frac{DF(t_{i-1})/DF(t_i)-1}{t_i - t_{i-1}}$	-
Clean Price	$\frac{PV}{N} \cdot 100$	%
Dirty Price	Clean Price $+\frac{AI}{N} \cdot 100$	%
Macaulay Duration	$\frac{1}{PV} \sum t_i \cdot CF_i \cdot DF(t_i)$	years
Modified Duration	$\frac{D_{Mac}}{1+r/f}$	years
DV01	$-(PV_{r+1bp} - PV_r)$	EUR
Convexity	$\frac{PV^+ + PV^- - 2PV}{PV \cdot (\Delta r)^2}$	-
Parametric VaR	$z_\alpha \cdot \sigma \cdot \sqrt{h/250} \cdot 10000 \cdot \lvert DV01 \rvert$	EUR
Portfolio VaR	$z_\alpha \cdot \sqrt{\mathbf{w}^T \mathbf{\Sigma} \mathbf{w}} \cdot \sqrt{h/250} \cdot 10000$	EUR
Covariance	$\frac{1}{T-1} \sum (\Delta r_{i,t} - \bar{\Delta r_i})(\Delta r_{j,t} - \bar{\Delta r_j})$	-
Correlation	$\frac{\Sigma_{ij}}{\sigma_i \cdot \sigma_j}$	-

14 References

- Hull, J. C. (2018). *Options, Futures, and Other Derivatives*. Pearson.
- Tuckman, B., & Serrat, A. (2011). *Fixed Income Securities*. Wiley.
- Fabozzi, F. J. (2012). *Bond Markets, Analysis, and Strategies*. Pearson.
- Jorion, P. (2006). *Value at Risk: The New Benchmark for Managing Financial Risk*. McGraw-Hill.
- Alexander, C. (2008). *Market Risk Analysis, Volume IV: Value at Risk Models*. Wiley.