Exercise Collection

Bond Valuation and Risk Calculator

English

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Notes for Working Through the Exercises

Diese Exercise Collection dient zur Vertiefung der Kenntnisse in der Bond-Bewertung und im Risikomanagement. Die Aufgaben sind nach Schwierigkeitsgrad und Themengebiet gegliedert:

- **Easy:** Basic calculations using given formulae
- **Medium:** Multi-step calculations and interpretations
- **Hard:** Complex problems and portfolio analyses

Recommended tools:

- Calculator or Excel/Python
- Formula sheet from the documentation
- Bond Calculator v3 for verification

Note: All calculations use continuous discounting ($e^{-r \cdot t}$) unless stated otherwise.

1 Fundamentals of Discounting

1.1 Exercise 1.1: Simple Discounting (Easy)

A bond pays exactly EUR 1 000 in 3 years. The discount rate from the yield curve is a constant 4% p.a.

Required:

a) Calculate the discount factor for 3 years.

b) Calculate the present value of this payment.

c) What would the present value be at a discount rate of 5%?

1.2 Exercise 1.2: Yield Curve Interpolation (Medium)

The following yield curve (discount rates) is given:

- 1 year: 2.5%
- 3 years: 3.5%

Required:

a) Calculate the discount rate for 2 years by linear interpolation.

b) Calculate the discount factor for 2 years.

c) A cash flow of EUR 500 falls due in 2 years. What is its present value?

1.3 Exercise 1.3: Multiple Discounting (Medium)

An investment pays:

- EUR 100 in 1 year
- EUR 150 in 2 years
- EUR 200 in 3 years

The yield curve (discount rates) is flat at 3% p.a.

Required:

a) Calculate the present value of all three payments.

b) How does the total PV change if all discount rates rise by 1%?

c) Calculate the percentage change in PV.

2 Fixed Rate Bonds

2.1 Exercise 2.1: Basic Valuation (Easy)

A fixed rate bond has the following parameters:

- Nominal amount: EUR 1 000 000
- Coupon: 4% p.a.
- Term: 3 years
- Payment frequency: annual
- Discount rate (yield curve): constant 3.5%

Required:

a) What is the annual coupon payment?

b) Calculate the present value of all coupon payments.

c) Calculate the present value of the redemption.

d) What is the total present value of the bond?

e) Calculate the clean price as a percentage.

2.2 Exercise 2.2: Semi-Annual Payments (Medium)

A corporate bond has the following characteristics:

- Nominal amount: EUR 500 000
- Coupon: 5.5% p.a.
- Term: 4 years
- Payment frequency: semi-annual
- Zinskurve: 0.5Y: 4%, 1Y: 4.2%, 2Y: 4.5%, 3Y: 4.7%, 4Y: 5%

Required:

a) How many coupon payments are there in total?

b) What is each individual coupon payment?

c) Calculate the interest rate for 0.5 years, 1.5 years and 2.5 years by linear interpolation.

d) Calculate the first three discounted coupon payments.

e) Estimate the total present value (without calculating all terms in full).

2.3 Exercise 2.3: Accrued Interest (Medium)

A bond with semi-annual payments has:

- Coupon: 6% p.a.
- Nominal amount: EUR 1 000 000
- Last coupon payment was 45 days ago

Required:

a) How many days does a coupon period have?

b) Calculate the accrued interest.

c) If the clean price is 102%, what is the dirty price?

2.4 Exercise 2.4: Sensitivity to Interest Rate Changes (Hard)

A 5-year bond with a 4% coupon (annual) and face value of EUR 1 000 000 is valued at a constant discount rate of 4%.

Required:

a) Calculate the present value at r = 4%.

b) Calculate the present value at r = 4.01% (increase of 1bp).

c) Calculate the present value at r = 3.99% (decrease of 1bp).

d) Calculate DV01 (Dollar Value of 01).

e) Calculate the convexity of the bond.

f) Estimate the price change for a rate increase of 50bp using DV01.

3 Floating Rate Bonds

3.1 Exercise 3.1: Forward Rate Calculation (Medium)

The following yield curve is given:

- 3 months (0.25Y): 2.0%
- 6 months (0.5Y): 2.2%

Required:

a) Calculate the discount factor for 3 months.

b) Calculate the discount factor for 6 months.

c) Calculate the forward rate for the second period (from 3 to 6 months).

d) Interpret the result: is the forward rate higher or lower than the spot rate at 6 months?

3.2 Exercise 3.2: Floating Rate Bond Valuation (Hard)

A floating rate bond has the following parameters:

- Nominal amount: EUR 2 000 000
- Spread: 100 basis points (1%)
- Term: 2 years
- Payment frequency: quarterly
- Zinskurve: 3M: 2.5%, 6M: 2.6%, 9M: 2.7%, 12M: 2.8%, 18M: 3.0%, 24M: 3.2%

Required:

a) How many payments are there in total?

b) Calculate the forward rate for the first period (0–3M).

c) Calculate the forward rate for the second period (3M–6M).

d) Calculate the coupon payment for the first period (forward rate + spread).

e) Calculate the coupon payment for the second period.

f) Calculate the present value of the first two coupon payments.

g) Why does this bond probably trade above or below par?

3.3 Exercise 3.3: Spread Analysis (Medium)

Two floating rate bonds with identical parameters except:

- Bond A: Spread = 50bp
- Bond B: Spread = 150bp

Both bonds have a face value of EUR 1 000 000 and a 3-year term with semi-annual payments.

Required:

a) By how many EUR do the coupon payments differ per period?

b) Estimate the difference in present value of the two bonds (qualitatively).

c) Which factors justify a higher spread?

4 Zero-Coupon Bonds

4.1 Exercise 4.1: Basic Valuation (Easy)

A zero-coupon bond has:

- Nominal amount / redemption: EUR 1 000 000
- Remaining term: 5 years
- Discount rate (yield curve): 3%

Required:

a) Calculate the present value.

b) Calculate the clean price as a percentage.

c) What is the dirty price? Explain.

d) What is the Macaulay Duration?

e) What is the Modified Duration?

4.2 Exercise 4.2: Yield Calculation (Medium)

A zero-coupon bond with 10 years remaining term trades at 74.08% of face value.

Required:

a) Set up the equation for the implied discount rate (yield).

b) Solve the equation: $0.7408 = e^{-r \cdot 10}$

c) Calculate the yield (implied discount rate).

d) What would the price be at a discount rate of 4%?

4.3 Exercise 4.3: Comparison with Coupon Bond (Hard)

Two bonds are given with identical term of 5 years and face value of EUR 1 000 000:

- Bond A: Zero-coupon bond
- Bond B: Fixed rate bond with 5% coupon (annual)

Both are valued at the same discount rate of 5%.

Required:

a) Calculate the PV of Bond A.

b) Calculate the PV of Bond B.

c) Which bond has the higher duration? Justify without calculation.

d) Which bond is more sensitive to interest rate changes?

e) Calculate DV01 for both bonds.

5 Risk Metrics

5.1 Exercise 5.1: Macaulay Duration (Medium)

A 3-year bond with a 6% coupon (annual) and face value of EUR 1 000 is valued at a constant discount rate of 5%.

Required:

a) Calculate the three coupon payments.

b) Calculate the present value of each payment (including redemption).

c) Calculate the total present value.

d) Calculate the weighted maturities: $t_i \cdot PV_i$

e) Calculate the Macaulay Duration.

f) Interpret the result.

5.2 Exercise 5.2: Modified Duration and Price Change (Hard)

A bond has the following metrics:

- Present value: EUR 1 050 000
- Macaulay Duration: 4.5 years
- Average discount rate: 4%
- Payment frequency: semi-annual

Required:

a) Calculate the Modified Duration.

b) Estimate the price change for a rate increase of 50bp (0.5%) using duration.

c) Calculate the new estimated price.

d) Explain why this is only an approximation.

5.3 Exercise 5.3: DV01 Portfolio (Hard)

A portfolio consists of:

- EUR 10m Bond A with DV01 = EUR 4 500
- EUR 5m Bond B with DV01 = EUR 2 800
- EUR 15m Bond C with DV01 = EUR 6 200

Required:

a) Calculate the portfolio DV01.

b) What is the estimated loss for a parallel rate rise of 25bp?

c) How much EUR must be invested in a bond with DV01 = EUR 3 000 to hedge the portfolio?

5.4 Exercise 5.4: Convexity (Hard)

A bond is valued at r = 4%:

- $PV(r = 4.00\%) = \text{EUR } 1\,000\,000$
- $PV(r = 4.01\%) = \text{EUR } 999\,550$
- $PV(r = 3.99\%) = \text{EUR } 1\,000\,455$

Required:

a) Calculate the convexity using the formula.

b) Interpret: what does positive convexity mean?

c) Estimate the price change for a rate increase of 100bp if the Modified Duration = 7 years.

d) Improve the estimate by accounting for convexity.

6 Parametric Value at Risk

6.1 Exercise 6.1: Basic VaR Calculation (Easy)

A bond has:

- DV01 = EUR 5 000
- Daily interest rate volatility: 8 basis points
- Confidence level: 95% ($z = 1.645$)
- Time horizon: 1 day

Required:

a) Calculate the 1-day VaR.

b) Interpret the result.

c) How does VaR change at 99% confidence ($z = 2.326$)?

6.2 Exercise 6.2: VaR Scaling (Medium)

Gegeben:

- DV01 = EUR 8 500
- Daily volatility: 10bp
- Konfidenzniveau: 95%
- Time horizon: 10 days

Required:

a) Calculate the scaled volatility for 10 days.

b) Calculate the 10-day VaR.

c) Why is VaR not simply multiplied by 10?

d) Calculate the VaR for 1 month (approx. 20 trading days).

6.3 Exercise 6.3: VaR Comparison (Medium)

Two bonds have identical PV of EUR 10m, but:

- Bond A: DV01 = EUR 5 000, Duration = 5 Jahre
- Bond B: DV01 = EUR 10 000, duration = 10 years

With identical volatility of 8bp and 95% confidence (1 day):
Required:

a) Calculate VaR for both bonds.

b) Which bond is riskier?

c) Explain the relationship between duration and VaR.

6.4 Exercise 6.4: VaR Portfolio (Hard)

A portfolio of 3 bonds (simplified assumption: perfect correlation):

- Bond 1: DV01 = EUR 4 000
- Bond 2: DV01 = EUR 6 000
- Bond 3: DV01 = EUR 5 000

Volatility: 7bp, 95% confidence, 5 days:
Required:

a) Calculate the portfolio DV01.

b) Calculate the scaled volatility.

c) Calculate the portfolio VaR.

d) How would VaR change if the correlation were not perfect? (qualitatively)

7 Empirical Portfolio VaR

7.1 Exercise 7.1: Returns Calculation (Easy)

Historical interest rates for the 2-year maturity:

- Tag 1: 3.50%
- Tag 2: 3.48%
- Tag 3: 3.52%
- Tag 4: 3.49%

Required:

a) Calculate the returns (differences) for days 2, 3 and 4.

b) Calculate the mean of the returns.

c) Calculate the variance of the returns.

d) Calculate the standard deviation (volatility) in basis points.

7.2 Exercise 7.2: Covariance Calculation (Medium)

Historical returns for two maturities (3 observations):

Day	2Y Return	5Y Return
1	-0.02%	-0.015%
2	+0.04%	+0.03%
3	-0.01%	-0.008%

Required:

a) Calculate the mean for both maturities.

b) Calculate the covariance between 2Y and 5Y returns.

c) Calculate the standard deviations for both.

d) Calculate the correlation between the two maturities.

e) Interpret the correlation.

7.3 Exercise 7.3: DV01 Vector (Medium)

A 5-year bond with a 4% coupon (semi-annual) is to be analysed for key rate risks.
The yield curve has nodes at 0.5Y, 1Y, 2Y, 3Y, 5Y.
Calculations give (at r=3%):

- PV (Basis) = 1,047,000 EUR
- PV (0.5Y +1bp) = 1,046,850 EUR

- PV (1Y +1bp) = 1,046,700 EUR

Required:

a) Calculate DV01 for the 0.5Y maturity.

b) Calculate DV01 for the 1Y maturity.

c) Which maturity has the greater influence on the bond value? Why?

7.4 Exercise 7.4: Portfolio VaR with Variance-Covariance Matrix (Hard)

A bond portfolio is analysed with 3 maturities: 1Y, 3Y, 5Y.

Given:

DV01 vector (in EUR):

$$\mathbf{w} = \begin{pmatrix} 500 \\ 1{,}200 \\ 800 \end{pmatrix}$$

Covariance matrix (in 10^{-6}):

$$\mathbf{\Sigma} = \begin{pmatrix} 9.0 & 7.5 & 5.0 \\ 7.5 & 8.0 & 6.5 \\ 5.0 & 6.5 & 7.0 \end{pmatrix}$$

Confidence level: 95% ($z = 1.645$), horizon: 10 days

Required:

a) Calculate $\mathbf{w}^T\mathbf{\Sigma}$ (row vector).

b) Calculate $\mathbf{w}^T\mathbf{\Sigma}\mathbf{w}$ (portfolio variance, daily basis).

c) Calculate the portfolio standard deviation (daily basis).

d) Scale to 10 days.

e) Calculate the portfolio VaR.

f) Interpret the result.

7.5 Exercise 7.5: Empirical vs. Parametric Method (Hard)

A bond is evaluated using both VaR methods:

Parametric:

- DV01 = 6,000 EUR
- Assumed volatility: 10bp/day
- 95% confidence, 5 days

Empirical:

- From 250 historical days: $\sigma = 12$bp/day
- Covariance with other maturities accounted for
- Portfolio variance = 0.000150 (after matrix multiplication)
- 95% confidence, 5 days

Required:

a) Calculate the parametric VaR.

b) Calculate the empirical VaR.

c) Compare both results. Which method is more conservative?

d) Which method is preferable in practice and why?

e) State the advantages and disadvantages of each approach.

8 Integrated Case Studies

8.1 Exercise 8.1: Bond Portfolio (Very Hard)

An institutional investor manages a portfolio of three bonds:

Bond A (Government):

- Nominal: EUR 10m
- Type: Fixed rate, 3% coupon, 5 years, annual
- Yield curve: flat at 2.5%

Bond B (Corporate):

- Nominal: EUR 5m
- Type: Fixed rate, 5% coupon, 3 years, semi-annual
- Yield curve: flat at 4%

Bond C (Floating):

- Nominal: EUR 8m
- Type: Floating rate, 120bp spread, 4 years, quarterly
- Zinskurve: 3M: 2.5%, 1Y: 2.8%, 2Y: 3.0%, 3Y: 3.2%, 4Y: 3.4%

Required:

a) Calculate the present value of each bond (rough estimate sufficient).

b) Calculate the total portfolio value.

c) Estimate the duration for each bond (qualitatively: short/medium/long).

d) Which bond is most sensitive to interest rate changes?

e) Bei einem DV01 von Bond A = 4,500 EUR, Bond B = 1,200 EUR, Bond C = 2,800 EUR: Calculate the portfolio DV01.

f) Estimate the loss for a parallel rate rise of 50bp.

g) What strategy would you recommend to reduce interest rate risk?

8.2 Exercise 8.2: Risk Management Decision (Very Hard)

You are a risk manager at a bank. A trader wishes to build the following portfolio:

- Long position: EUR 20m in 10-year government bonds (DV01 = EUR 9 000)
- Short position: EUR 15m in 2-year corporate bonds (DV01 = EUR 2 500)

Historical volatilities:

- 10Y: 5bp/day
- 2Y: 8bp/day
- Correlation: 0.7

The bank's VaR limit is EUR 100 000 (95%, 10 days).
Required:

a) Calculate the net DV01 of the portfolio.

b) Is the portfolio long or short rates?

c) Estimate the parametric VaR (simplified: weighted average volatility).

d) Is the VaR limit respected?

e) What adjustments would you suggest?

f) How would VaR change if the correlation were only 0.3?

9 Solutions

Solution 1.1: Simple Discounting

a) Discount factor:

$$DF(3) = e^{-0.04 \cdot 3} = e^{-0.12} = 0.8869$$

b) Present Value:

$$PV = 1{,}000 \cdot 0.8869 = 886.90 \text{ EUR}$$

c) Present Value at 5%:

$$DF(3) = e^{-0.05 \cdot 3} = e^{-0.15} = 0.8607$$

$$PV = 1{,}000 \cdot 0.8607 = 860.70 \text{ EUR}$$

Interpretation: At higher discount rates the present value falls.

Solution 1.2: Yield Curve Interpolation

a) Discount rate for 2 years:

$$r(2) = 2.5\% + \frac{3.5\% - 2.5\%}{3 - 1} \cdot (2 - 1) = 2.5\% + 0.5\% = 3.0\%$$

b) Discount factor:

$$DF(2) = e^{-0.03 \cdot 2} = e^{-0.06} = 0.9418$$

c) Present Value:

$$PV = 500 \cdot 0.9418 = 470.90 \text{ EUR}$$

Solution 1.3: Multiple Discounting

a) Present value at 3%:

$$\begin{aligned} PV_1 &= 100 \cdot e^{-0.03 \cdot 1} = 97.04 \text{ EUR} \\ PV_2 &= 150 \cdot e^{-0.03 \cdot 2} = 141.27 \text{ EUR} \\ PV_3 &= 200 \cdot e^{-0.03 \cdot 3} = 177.38 \text{ EUR} \\ PV_{\text{total}} &= 415.69 \text{ EUR} \end{aligned}$$

b) Present value at 4%:

$$\begin{aligned} PV_1 &= 100 \cdot e^{-0.04 \cdot 1} = 96.08 \text{ EUR} \\ PV_2 &= 150 \cdot e^{-0.04 \cdot 2} = 138.45 \text{ EUR} \\ PV_3 &= 200 \cdot e^{-0.04 \cdot 3} = 176.70 \text{ EUR} \\ PV_{\text{total}} &= 411.23 \text{ EUR} \end{aligned}$$

c) Percentage change:

$$\frac{411.23 - 415.69}{415.69} \cdot 100\% = -1.07\%$$

Interpretation: A 1% increase in discount rates leads to a value loss of approx. 1%.

Solution 2.1: Fixed Rate Bond Basic Valuation

a) Coupon payment:

$$C = \frac{1{,}000{,}000 \cdot 0.04}{1} = 40{,}000 \text{ EUR}$$

b) PV of all coupon payments:

$$\begin{aligned} PV_{\text{Kupons}} &= 40{,}000 \cdot e^{-0.035 \cdot 1} + 40{,}000 \cdot e^{-0.035 \cdot 2} + 40{,}000 \cdot e^{-0.035 \cdot 3} \\ &= 38{,}617 + 37{,}273 + 35{,}965 \\ &= 111{,}855 \text{ EUR} \end{aligned}$$

c) PV of redemption:

$$PV_{\text{Rückz}} = 1{,}000{,}000 \cdot e^{-0.035 \cdot 3} = 899{,}122 \text{ EUR}$$

d) Total PV:

$$PV_{\text{total}} = 111{,}855 + 899{,}122 = 1{,}010{,}977 \text{ EUR}$$

e) Clean Price:

$$\text{Clean Price} = \frac{1{,}010{,}977}{1{,}000{,}000} \cdot 100\% = 101.10\%$$

Interpretation: The bond trades above par (100%) because the coupon (4%) exceeds the discount rate (3.5%).

Solution 2.2: Semi-Annual Payments

a) Number of payments:

$$n = 4 \text{ Jahre} \times 2 = 8 \text{ Zahlungen}$$

b) Coupon payment:

$$C = \frac{500{,}000 \cdot 0.055}{2} = 13{,}750 \text{ EUR}$$

c) Interpolated rates:

$$\begin{aligned} r(0.5) &= 4.0\% \text{ (gegeben)} \\ r(1.5) &= 4.2\% + \frac{4.5\% - 4.2\%}{2 - 1} \cdot 0.5 = 4.35\% \\ r(2.5) &= 4.5\% + \frac{4.7\% - 4.5\%}{3 - 2} \cdot 0.5 = 4.6\% \end{aligned}$$

d) First three discounted coupon payments:

$$\begin{aligned} PV_1 &= 13{,}750 \cdot e^{-0.04 \cdot 0.5} = 13{,}479 \text{ EUR} \\ PV_2 &= 13{,}750 \cdot e^{-0.042 \cdot 1.0} = 13{,}186 \text{ EUR} \\ PV_3 &= 13{,}750 \cdot e^{-0.0435 \cdot 1.5} = 12{,}874 \text{ EUR} \end{aligned}$$

e) Estimated total PV:

The 8 coupon payments sum to approx. EUR 105 000 (PV). The redemption: $500{,}000 \cdot e^{-0.05 \cdot 4} \approx 409{,}365$ EUR Estimated total PV: approx. EUR 514 000

Interpretation: The bond trades above par (approx. 103%) because the coupon (5.5%) exceeds the discount rates (4–5%).

Solution 2.3: Accrued Interest

a) Days per period:

$$d_{\text{Periode}} = \frac{365}{2} = 182.5 \text{ Tage}$$

b) Accrued Interest:

$$AI = \frac{1{,}000{,}000 \cdot 0.06}{2} \cdot \frac{45}{182.5} = 30{,}000 \cdot 0.2466 = 7{,}397 \text{ EUR}$$

c) Dirty Price:

$$\text{Dirty Price} = 102\% + \frac{7{,}397}{1{,}000{,}000} \cdot 100\% = 102.74\%$$

Solution 2.4: Sensitivity to Interest Rate Changes

a) PV at r=4%:

At r=4% (discount rate) and 4% coupon the bond trades at par:

$$PV = 1{,}000{,}000 \text{ EUR}$$

b) PV at r=4.01%:

$$\begin{aligned} PV^{+} &= \sum_{i=1}^{5} 40{,}000 \cdot e^{-0.0401 \cdot i} + 1{,}000{,}000 \cdot e^{-0.0401 \cdot 5} \\ &\approx 999{,}554 \text{ EUR} \end{aligned}$$

c) PV at r=3.99%:

$$PV^{-} \approx 1{,}000{,}446 \text{ EUR}$$

d) DV01:

$$DV01 = -(PV^{+} - PV) = -(999{,}554 - 1{,}000{,}000) = 446 \text{ EUR}$$

e) Konvexität:

$$\text{Konvexität} = \frac{PV^{+} + PV^{-} - 2 \cdot PV}{PV \cdot (0.0001)^2} = \frac{999{,}554 + 1{,}000{,}446 - 2{,}000{,}000}{1{,}000{,}000 \cdot 0.00000001} = 0$$

(Convexity is very low for a par bond with a flat curve)

f) Price change at +50bp:

$$\Delta PV \approx -DV01 \cdot 50 = -446 \cdot 50 = -22{,}300 \text{ EUR}$$

New estimated price: EUR 977 700

Solution 3.1: Forward Rate Calculation

a) DF for 3 months:

$$DF(0.25) = e^{-0.02 \cdot 0.25} = e^{-0.005} = 0.9950$$

b) DF for 6 months:

$$DF(0.5) = e^{-0.022 \cdot 0.5} = e^{-0.011} = 0.9891$$

c) Forward Rate:

$$f_2 = \frac{DF(0.25)/DF(0.5) - 1}{0.5 - 0.25} = \frac{0.9950/0.9891 - 1}{0.25} = \frac{1.00596 - 1}{0.25} = 0.0238 = 2.38\%$$

d) Interpretation:
The forward rate (2.38%) is higher than the spot rate at 3M (2.0%) but lower than at 6M (2.2%). This reflects the upward-sloping yield structure.

Solution 3.2: Floating Rate Bond (abridged)

a) 8 Zahlungen (2 Jahre × 4)
b) Forward Rate Periode 1: $f_1 = 2.5\%$ (Spot Rate)
c) Forward Rate Periode 2:

$$f_2 = \frac{e^{-0.025 \cdot 0.25}/e^{-0.026 \cdot 0.5} - 1}{0.25} \approx 2.7\%$$

d) Kupon Periode 1:

$$C_1 = \frac{2{,}000{,}000 \cdot (0.025 + 0.01)}{4} = 17{,}500 \text{ EUR}$$

e) Kupon Periode 2:

$$C_2 = \frac{2{,}000{,}000 \cdot (0.027 + 0.01)}{4} = 18{,}500 \text{ EUR}$$

f) PV der ersten beiden:

$$\begin{aligned} PV &= 17{,}500 \cdot e^{-0.025 \cdot 0.25} + 18{,}500 \cdot e^{-0.026 \cdot 0.5} \\ &\approx 17{,}391 + 18{,}263 = 35{,}654 \text{ EUR} \end{aligned}$$

g) The bond trades above par because the spread (100bp) is paid in addition to the forward rates.

Solution 4.1: Zero-Coupon Bond

a) Present Value:

$$PV = 1{,}000{,}000 \cdot e^{-0.03 \cdot 5} = 860{,}708 \text{ EUR}$$

b) Clean Price:

$$\text{Clean Price} = \frac{860{,}708}{1{,}000{,}000} \cdot 100\% = 86.07\%$$

c) Dirty Price:
For zero-coupon bonds: Clean Price = Dirty Price = 86.07%, since no accrued interest arises.

d) Macaulay Duration:
For zero-coupon bonds the duration equals the term:

$$D_{\text{Mac}} = 5 \text{ Jahre}$$

e) Modified Duration:

$$D_{\text{Mod}} = \frac{5}{1 + 0.03} = 4.85 \text{ Jahre}$$

Solution 4.2: Yield Calculation

a) Equation:

$$0.7408 = e^{-r \cdot 10}$$

b) Solution:

$$\ln(0.7408) = -r \cdot 10$$
$$-0.3001 = -r \cdot 10$$
$$r = 0.03001 = 3.00\%$$

c) The yield (implied discount rate) is 3%.
d) Price at 4% discount rate:

$$\text{Preis} = e^{-0.04 \cdot 10} = 0.6703 = 67.03\%$$

Solution 5.1: Macaulay Duration

a) Coupon payments: $C = \text{EUR } 60$ (annual for 3 years)
b) Present Values:

$$PV_1 = 60 \cdot e^{-0.05 \cdot 1} = 57.07 \text{ EUR}$$
$$PV_2 = 60 \cdot e^{-0.05 \cdot 2} = 54.27 \text{ EUR}$$
$$PV_3 = 1{,}060 \cdot e^{-0.05 \cdot 3} = 913.03 \text{ EUR}$$

c) Total PV:

$$PV = 1{,}024.37 \text{ EUR}$$

d) Weighted maturities:

$$1 \cdot 57.07 = 57.07$$
$$2 \cdot 54.27 = 108.54$$
$$3 \cdot 913.03 = 2{,}739.09$$
$$\text{Summe: } = 2{,}904.70$$

e) Macaulay Duration:

$$D_{\text{Mac}} = \frac{2{,}904.70}{1{,}024.37} = 2.84 \text{ Jahre}$$

f) Interpretation:
On average the capital is committed for 2.84 years. This is shorter than the 3-year term because coupon payments return earlier.

Solution 6.1: Basic VaR

a) 1-day VaR:

$$VaR = 1.645 \cdot 0.0008 \cdot 10{,}000 \cdot 5{,}000 = 65{,}800 \text{ EUR}$$

b) Interpretation:
With 95% probability the loss on any single day will not exceed EUR 65 800.
c) VaR at 99%:

$$VaR_{99} = 2.326 \cdot 0.0008 \cdot 10{,}000 \cdot 5{,}000 = 93{,}040 \text{ EUR}$$

Solution 7.4: Portfolio VaR with Matrix (detailed)

a) Calculate $\mathbf{w}^T\boldsymbol{\Sigma}$:

$$\mathbf{w}^T\boldsymbol{\Sigma} = (500 \quad 1{,}200 \quad 800) \begin{pmatrix} 9.0 & 7.5 & 5.0 \\ 7.5 & 8.0 & 6.5 \\ 5.0 & 6.5 & 7.0 \end{pmatrix}$$

First component: $500 \cdot 9.0 + 1{,}200 \cdot 7.5 + 800 \cdot 5.0 = 4{,}500 + 9{,}000 + 4{,}000 = 17{,}500$
Second component: $500 \cdot 7.5 + 1{,}200 \cdot 8.0 + 800 \cdot 6.5 = 3{,}750 + 9{,}600 + 5{,}200 = 18{,}550$
Third component: $500 \cdot 5.0 + 1{,}200 \cdot 6.5 + 800 \cdot 7.0 = 2{,}500 + 7{,}800 + 5{,}600 = 15{,}900$

$$\mathbf{w}^T\boldsymbol{\Sigma} = (17{,}500 \quad 18{,}550 \quad 15{,}900)$$

b) Calculate $\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}$:

$$\sigma_P^2 = 17{,}500 \cdot 500 + 18{,}550 \cdot 1{,}200 + 15{,}900 \cdot 800$$
$$= 8{,}750{,}000 + 22{,}260{,}000 + 12{,}720{,}000 = 43{,}730{,}000 \times 10^{-6}$$
$$= 0.04373$$

c) Portfolio standard deviation:

$$\sigma_P = \sqrt{0.04373} = 0.2091$$

(This corresponds to approx. 209bp or 0.02091 in decimal form per day)
d) Scaling to 10 days:

$$\sigma_{P,10} = 0.2091 \cdot \sqrt{\frac{10}{250}} = 0.2091 \cdot 0.2 = 0.04182$$

e) Portfolio VaR:

$$VaR = 1.645 \cdot 0.04182 \cdot 10{,}000 = 687.95 \text{ EUR}$$

f) Interpretation:
With 95% probability the portfolio loss over 10 days will not exceed EUR 688. This accounts for both individual volatilities and the correlations between maturities.

Solution 2.3: Accrued Interest

a) Days per period:

$$d_{\text{Periode}} = \frac{365}{2} = 182.5 \text{ Tage}$$

b) Accrued Interest:

$$AI = \frac{1{,}000{,}000 \cdot 0.06}{2} \cdot \frac{45}{182.5} = 30{,}000 \cdot 0.2466 = 7{,}397 \text{ EUR}$$

c) Dirty Price:

$$\text{Dirty Price} = 102\% + \frac{7{,}397}{1{,}000{,}000} \cdot 100\% = 102.74\%$$

Solution 3.3: Spread Analysis

a) Difference in coupon payments:
Kupon pro Periode bei Bond A: $\frac{1{,}000{,}000 \cdot (f+0.005)}{2}$
Kupon pro Periode bei Bond B: $\frac{1{,}000{,}000 \cdot (f+0.015)}{2}$
Unterschied: $\frac{1{,}000{,}000 \cdot 0.01}{2} = 5{,}000$ EUR pro Periode
b) Difference in present value:
Bond B pays EUR 5 000 more in each period. With 6 periods over 3 years and a discount rate of approx. 3%:

$$\Delta PV \approx 6 \cdot 5{,}000 \cdot e^{-0.03 \cdot 1.5} \approx 28{,}500 \text{ EUR}$$

Bond B is worth approx. EUR 28 500 more.
c) Factors justifying a higher spread:

- Higher credit risk (lower credit quality)
- Lower liquidity
- Longer term
- Smaller issuer
- Industry-specific risks

Solution 4.3: Comparison with Coupon Bond

a) PV of Bond A (zero-coupon):

$$PV_A = 1{,}000{,}000 \cdot e^{-0.05 \cdot 5} = 778{,}801 \text{ EUR}$$

b) PV of Bond B (5% coupon):
Since coupon = discount rate = 5%, the bond trades at par:

$$PV_B = 1{,}000{,}000 \text{ EUR}$$

c) Higher Duration:
Bond A has the higher duration (5 years) because:

- Zero-coupon: Duration = term = 5 years
- Coupon bond: Duration ¡ term, since cash flows return earlier

d) Sensitivity:
Bond A is more sensitive, since higher duration → higher interest rate risk
e) DV01 calculation:
Bond A:

$$PV(5.01\%) = 1{,}000{,}000 \cdot e^{-0.0501 \cdot 5} = 778{,}412 \text{ EUR}$$
$$DV01_A = 778{,}801 - 778{,}412 = 389 \text{ EUR}$$

Bond B: Bei r=5%: PV = 1,000,000 EUR
Bei r=5.01%:

$$PV \approx 999{,}520 \text{ EUR}$$
$$DV01_B \approx 480 \text{ EUR}$$

Bond B has a higher DV01 despite lower duration because the nominal value is higher (1m vs. 779k).

Solution 5.2: Modified Duration and Price Change

a) Modified Duration:

$$D_{\text{Mod}} = \frac{4.5}{1 + 0.04} = \frac{4.5}{1.04} = 4.327 \text{ Jahre}$$

b) Price change at +50bp:

$$\Delta PV \approx -D_{\text{Mod}} \cdot \Delta r \cdot PV = -4.327 \cdot 0.005 \cdot 1{,}050{,}000$$
$$= -22{,}717 \text{ EUR}$$

c) New estimated price:

$$PV_{\text{neu}} = 1{,}050{,}000 - 22{,}717 = 1{,}027{,}283 \text{ EUR}$$

d) Why only an approximation?
The duration approximation is linear and ignores the convexity (curvature) of the price-yield relationship. For larger rate changes the error becomes significant. The actual price-yield relationship is convex, meaning:

- Rate decrease: actual gain ¿ duration estimate
- Rate increase: actual loss ¡ duration estimate

Solution 5.3: DV01 Portfolio

a) Portfolio DV01:

$$DV01_{\text{Portfolio}} = 4{,}500 + 2{,}800 + 6{,}200 = 13{,}500 \text{ EUR}$$

b) Loss at +25bp:

$$\text{Verlust} = DV01 \cdot 25 = 13{,}500 \cdot 25 = 337{,}500 \text{ EUR}$$

c) Hedge amount:
To hedge the portfolio we need:

$$DV01_{\text{Hedge}} = -13{,}500 \text{ EUR}$$

For a bond with DV01 = EUR 3 000 per million:

$$\text{Nominalbetrag} = \frac{-13{,}500}{3{,}000} \cdot 1{,}000{,}000 = -4{,}500{,}000 \text{ EUR}$$

One must go **short** EUR 4.5m (sell).

Solution 5.4: Convexity

a) Convexity:

$$\begin{aligned}\text{Konvexität} &= \frac{PV^+ + PV^- - 2 \cdot PV}{PV \cdot (\Delta r)^2} \\ &= \frac{999{,}550 + 1{,}000{,}455 - 2{,}000{,}000}{1{,}000{,}000 \cdot (0.0001)^2} \\ &= \frac{5}{1{,}000{,}000 \cdot 0.00000001} = \frac{5}{0.00001} = 500{,}000\end{aligned}$$

b) Interpretation:
Positive convexity means:

- The price-yield curve curves upwards
- Rate decrease: price rises more than duration estimate
- Rate increase: price falls less than duration estimate
- This is advantageous for the bondholder

c) Duration estimate at +100bp:

$$\Delta PV_{\text{Duration}} = -7 \cdot 0.01 \cdot 1{,}000{,}000 = -70{,}000 \text{ EUR}$$

d) Improvement with convexity:

$$\Delta PV_{\text{Konvexität}} = \frac{1}{2} \cdot 500{,}000 \cdot (0.01)^2 \cdot 1{,}000{,}000 = 25{,}000 \text{ EUR}$$

$$\Delta PV_{\text{gesamt}} = -70{,}000 + 25{,}000 = -45{,}000 \text{ EUR}$$

The actual loss is only EUR 45 000 instead of EUR 70 000 – convexity reduces the loss.

Solution 6.2: VaR Scaling

a) Scaled volatility for 10 days:

$$\sigma_{10} = 10 \text{ bp} \cdot \sqrt{\frac{10}{250}} = 10 \cdot 0.2 = 2 \text{ bp}$$

b) 10-day VaR:

$$VaR_{10} = 1.645 \cdot 0.0002 \cdot 10{,}000 \cdot 8{,}500 = 27{,}965 \text{ EUR}$$

c) Why not simply $\times$ 10?
VaR scales with the square root of time, not linearly:

$$VaR_t = VaR_1 \cdot \sqrt{t}$$

This is because:

- Volatility grows with $\sqrt{t}$ (random walk)
- Positive and negative movements can partially cancel out
- Linear scaling would overestimate the risk

d) VaR for 1 month (20 days):

$$\sigma_{20} = 10 \text{ bp} \cdot \sqrt{\frac{20}{250}} = 10 \cdot \sqrt{0.08} = 2.828 \text{ bp}$$

$$VaR_{20} = 1.645 \cdot 0.00028828 \cdot 10{,}000 \cdot 8{,}500 = 40{,}239 \text{ EUR}$$

Solution 6.3: VaR Comparison

a) VaR for both bonds:
Bond A:

$$VaR_A = 1.645 \cdot 0.0008 \cdot 10{,}000 \cdot 5{,}000 = 65{,}800 \text{ EUR}$$

Bond B:

$$VaR_B = 1.645 \cdot 0.0008 \cdot 10{,}000 \cdot 10{,}000 = 131{,}600 \text{ EUR}$$

b) Riskier bond:
Bond B is twice as risky as Bond A (VaR is twice as high).
c) Relationship between Duration and VaR:

- Higher duration $\rightarrow$ higher DV01 $\rightarrow$ higher VaR
- Duration measures interest rate sensitivity
- VaR quantifies the resulting loss risk
- At equal face value: VaR $\propto$ Duration

Solution 6.4: VaR Portfolio

a) Portfolio DV01:

$$DV01_P = 4{,}000 + 6{,}000 + 5{,}000 = 15{,}000 \text{ EUR}$$

b) Skalierte Volatilität:

$$\sigma_5 = 7 \text{ bp} \cdot \sqrt{\frac{5}{250}} = 7 \cdot \sqrt{0.02} = 0.9899 \text{ bp}$$

c) Portfolio-VaR:

$$VaR = 1.645 \cdot 0.000099 \cdot 10{,}000 \cdot 15{,}000 = 24{,}433 \text{ EUR}$$

d) Effect of imperfect correlation:
With imperfect correlation (e.g. $\rho = 0.7$) VaR would **fall**:

- Diversification effect: not all bonds move simultaneously
- Losses partially offset each other
- Portfolio VaR $<$ sum of individual VaRs
- The lower the correlation, the stronger the diversification effect

With perfect correlation ($\rho = 1$) there is no diversification effect.

Solution 7.1: Returns Calculation

a) Returns:

$$\begin{aligned}
\Delta r_2 &= 3.48\% - 3.50\% = -0.02\% = -2 \text{ bp} \\
\Delta r_3 &= 3.52\% - 3.48\% = +0.04\% = +4 \text{ bp} \\
\Delta r_4 &= 3.49\% - 3.52\% = -0.03\% = -3 \text{ bp}
\end{aligned}$$

b) Mean:

$$\overline{\Delta r} = \frac{-0.02 + 0.04 - 0.03}{3} = \frac{-0.01}{3} = -0.0033\% \approx -0.33 \text{ bp}$$

c) Variance:

$$\begin{aligned}
\sigma^2 &= \frac{1}{3-1} \sum (\Delta r_i - \overline{\Delta r})^2 \\
&= \frac{1}{2}[(-0.02 + 0.0033)^2 + (0.04 + 0.0033)^2 + (-0.03 + 0.0033)^2] \\
&= \frac{1}{2}[0.000278 + 0.001878 + 0.000711] = 0.001434
\end{aligned}$$

d) Standard deviation:

$$\sigma = \sqrt{0.001434} = 0.0379\% = 3.79 \text{ bp}$$

Solution 7.2: Covariance Calculation

a) Means:

$$\overline{\Delta r_{2Y}} = \frac{-0.02 + 0.04 - 0.01}{3} = 0.0033\%$$
$$\overline{\Delta r_{5Y}} = \frac{-0.015 + 0.03 - 0.008}{3} = 0.0023\%$$

b) Covariance:

$$Cov(2Y, 5Y) = \frac{1}{2}[(-0.0233)(-0.0173) + (0.0367)(0.0277) + (-0.0133)(-0.0103)]$$
$$= \frac{1}{2}[0.000405 + 0.001015 + 0.000139] = 0.000779$$

c) Standard deviations:

$$\sigma_{2Y} = 0.0300\%$$
$$\sigma_{5Y} = 0.0226\%$$

d) Correlation:

$$\rho = \frac{0.000779}{0.0300 \cdot 0.0226} = 0.988$$

e) Interpretation:
The correlation of 0.988 shows: 2Y and 5Y rates almost always move together. Typical for neighbouring maturities – little diversification.

Solution 7.3: DV01 Vector

a) DV01 für 0.5Y:

$$DV01_{0.5Y} = 1{,}047{,}000 - 1{,}046{,}850 = 150 \text{ EUR}$$

b) DV01 für 1Y:

$$DV01_{1Y} = 1{,}047{,}000 - 1{,}046{,}700 = 300 \text{ EUR}$$

c) Greater influence:
The 1Y maturity (DV01 = EUR 300) has greater influence, since more cash flows of a 5-year bond are affected by the 1Y rate.

Solution 7.5: Empirical vs. Parametric Method

a) Parametric VaR:

$$\sigma_5 = 10 \cdot \sqrt{5/250} = 1.414 \text{ bp}$$
$$VaR_{\text{param}} = 1.645 \cdot 0.00001414 \cdot 10{,}000 \cdot 6{,}000 = 13{,}961 \text{ EUR}$$

b) Empirical VaR:
Portfolio standard deviation (daily): $\sqrt{0.000150} = 0.01225 = 122.5$ bp
Scaled to 5 days:

$$\sigma_5 = 122.5 \cdot \sqrt{5/250} = 17.32 \text{ bp}$$

$$VaR_{\text{emp}} = 1.645 \cdot 0.0001732 \cdot 10{,}000 = 28{,}491 \text{ EUR}$$

c) Comparison:
The empirical method is more conservative (EUR 28 491 vs. EUR 13 961).

d) Preferred method:
Empirical method preferred: Based on actual market data, captures real volatilities and correlations, no normality assumption.

e) Advantages and disadvantages:
Parametric: Simple, few data required, but normality assumption often violated.
Empirical: No assumptions, realistic, but requires many data points (250+ days) and past $\neq$ future.

Solution 8.1: Bond Portfolio

a–b) Present Values:
Bond A (EUR 10m, 3% coupon, r=2.5%): $\approx$ EUR 10 220 000
Bond B (EUR 5m, 5% coupon, r=4%): $\approx$ EUR 5 140 000
Bond C (EUR 8m, Floating+120bp): $\approx$ EUR 8 080 000
Total: EUR 23 440 000
c) Duration: A: medium (4.5yr), B: short (2.7yr), C: very short (0.5yr)
d) Bond A most sensitive (highest duration)
e) Portfolio-DV01 = EUR 8 500
f) Loss at +50bp: EUR 425 000
g) Strategies:

- Duration matching by selling Bond A
- Interest rate swaps (pay-fixed)
- Short position in bond futures
- Reallocation to more floating rate

Solution 8.2: Risk Management Decision

a) Net DV01: 9 000 - 2 500 = EUR 6 500
b) Portfolio is **long rates** (positive DV01)
c) VaR (simplified):
Weighted volatility: 5.65bp
10-day VaR: EUR 12 088
d) Yes, limit respected (EUR 12 088 < EUR 100 000)
e) Adjustments:
Note basis risk (10Y govt vs. 2Y corp), spread risk and liquidity risk. Better matching with 10Y instruments or increasing the short position to DV01 = EUR 9 000.
f) At $\rho = 0.3$:
VaR would **fall** to approx. EUR 8 000–9 000 (stronger diversification effect at lower correlation).

Key Formulae for Review:

- Discount factor: $DF(t) = e^{-r(t)\cdot t}$
- Present value: $PV = \sum C_i \cdot DF(t_i) + N \cdot DF(t_n)$
- Forward rate: $f = \frac{DF(t_{i-1})/DF(t_i)-1}{t_i - t_{i-1}}$
- DV01: $DV01 = -(PV_{r+1\text{bp}} - PV_r)$
- Parametric VaR: $VaR = z_\alpha \cdot \sigma \cdot \sqrt{h/250} \cdot 10{,}000 \cdot |DV01|$
- Portfolio VaR: $VaR = z_\alpha \cdot \sqrt{\mathbf{w}^T \mathbf{\Sigma} \mathbf{w}} \cdot \sqrt{h/250} \cdot 10{,}000$

Closing Remarks

Diese Exercise Collection deckt die wichtigsten Aspekte der Bond-Bewertung und des Risikomanagements ab. Für tiefere Einblicke empfehlen wir:

- The mathematical documentation of Bond Calculator v3
- Practical exercises with Bond Calculator v3
- Specialist literature on fixed income and risk management

For questions or clarifications the accompanying documentation is available.