Gold Swap Calculator

Documentation with Calculation Procedures and Complete Practical Example

Student Edition – Quantitative Risk Management

English

Wolfgang Boehme

wolfgang.boehme@economic-modeling.de

www.economic-modeling.de

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This documentation describes the valuation and risk measurement of a Gold Swap using the Discount Factor method.

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Introduction

What is a Gold Swap?

A Gold Swap is a derivative contract in which two parties exchange cash flows:

- **EUR leg (fixed):** Fixed interest payments in EUR on a EUR notional
- **Gold leg:** Gold lease rate payments on the gold value, plus return of the gold value at maturity

Both legs are discounted with the EUR yield curve, since all cash flows are expressed in EUR.

Use Cases

1. **Hedging:** Protection against gold price risk
2. **Financing:** Gold-based financing structures
3. **Arbitrage:** Exploiting interest rate differentials
4. **Portfolio diversification:** Gold as an asset class

Risk Factors

With n payment dates the calculator uses $2n + 1$ risk factors:

- n EUR discount factors $DF_{\text{EUR}}(t_i)$
- n gold lease rates $L(t_i)$
- the gold price S_{Gold} in EUR per troy ounce

Mathematical Foundations

Discount Factors

Discount Factor

$$\mathrm{DF}(t) = e^{-r(t)\cdot t}$$

- $r(t)$: continuous zero rate for maturity t
- t: time in years

For payment dates between the curve points T_k of the EUR curve, interpolation works as in the swap and CCS calculators:

- $T_k < t < T_{k+1}$: linear in the rate

$$r(t) = r(T_k) + \frac{t - T_k}{T_{k+1} - T_k} \cdot [r(T_{k+1}) - r(T_k)]$$

- $t > T_m$ (beyond the last point): flat, $r(t) = r(T_m)$
- $t < T_1$ (before the first point): linear in the discount factor between $DF(0) = 1$ and $DF(T_1)$

Gold Lease Rate

The gold lease rate is the interest rate for borrowing physical gold. Rates of a few tenths of a percent p.a. are common; when gold liquidity is tight they can rise considerably. In the sample data they range from 0.32% to 0.96%.

Lease rates are interpolated linearly to the payment dates, flat at both ends. They are taken from the last row of the loaded time series.

Valuation Formulae

EUR Leg Present Value

EUR Leg PV

$$PV_{\text{EUR}} = \sum_{i=1}^{n} CF_{\text{EUR}}(t_i) \cdot DF_{\text{EUR}}(t_i)$$

with:

$$CF_{\text{EUR}}(t_i) = N_{\text{EUR}} \cdot r_{\text{EUR}} \cdot \Delta t + \mathbf{1}_{i=n} \cdot N_{\text{EUR}}$$

Gold Leg Present Value

Gold Leg PV

$$PV_{\text{Gold}} = \sum_{i=1}^{n} CF_{\text{Gold}}(t_i) \cdot DF_{\text{EUR}}(t_i)$$

with:

$$CF_{\text{Gold}}(t_i) = V_{\text{Gold}} \cdot L(t_i) \cdot \Delta t + \mathbf{1}_{i=n} \cdot V_{\text{Gold}}$$

$$V_{\text{Gold}} = N_{\text{oz}} \cdot S_{\text{Gold}}$$

Important: Both legs are discounted using the EUR yield curve.

Net Present Value

$$PV_{\text{net}} = \begin{cases} PV_{\text{Gold}} - PV_{\text{EUR}} & \text{Pay EUR} \\ PV_{\text{EUR}} - PV_{\text{Gold}} & \text{Pay Gold} \end{cases}$$

Sensitivities

Core Principle: Elasticity instead of Plain Derivative

Important Note

Sensitivities are computed as **elasticities**:

$$\boxed{\text{Sensitivity}_i = \frac{\partial PV_{\text{net}}}{\partial x_i} \cdot x_i = \frac{\partial PV_{\text{net}}}{\partial \ln x_i}}$$

Here x_i is the current value of risk factor i at the valuation date $t = 0$.

This definition is consistent with the use of **log returns** in the covariance matrix, since:

$$\Delta PV \approx \frac{\partial PV}{\partial x_i} \cdot \Delta x_i = \underbrace{\frac{\partial PV}{\partial x_i} \cdot x_i}_{\text{sensitivity}} \cdot \underbrace{\frac{\Delta x_i}{x_i}}_{\approx r_i}$$

The sensitivity is therefore the coefficient multiplying the dimensionless log return r_i. Rule of thumb: if the risk factor rises by 1%, the present value changes by about 1% of the sensitivity.

All formulae below apply to Pay EUR. For Pay Gold all signs are reversed.

EUR DF Sensitivities

The derivative of PV_{net} with respect to $DF_{\text{EUR}}(t_i)$ is:

$$\frac{\partial PV_{\text{net}}}{\partial DF_{\text{EUR}}(t_i)} = CF_{\text{Gold}}(t_i) - CF_{\text{EUR}}(t_i)$$

Multiplied by the current discount factor, this yields the **sensitivity**:

EUR DF Sensitivity (Pay EUR)

$$\Delta_{DF(t_i)} = [CF_{\text{Gold}}(t_i) - CF_{\text{EUR}}(t_i)] \cdot DF_{\text{EUR}}(t_i)$$

Gold Lease Rate Sensitivities

The derivative with respect to $L(t_i)$:

$$\frac{\partial PV_{\text{net}}}{\partial L(t_i)} = V_{\text{Gold}} \cdot \Delta t \cdot DF_{\text{EUR}}(t_i)$$

Lease Rate Sensitivity (Pay EUR)

$$\Delta_{L(t_i)} = V_{\text{Gold}} \cdot \Delta t \cdot DF_{\text{EUR}}(t_i) \cdot L(t_i)$$

Gold Spot Sensitivity

The derivative with respect to S_{Gold}:

$$\frac{\partial PV_{\text{net}}}{\partial S_{\text{Gold}}} = N_{\text{oz}} \cdot \left[\sum_{i=1}^{n} L(t_i) \cdot \Delta t \cdot DF_{\text{EUR}}(t_i) + DF_{\text{EUR}}(t_n) \right]$$

Gold Spot Sensitivity (Pay EUR)

$$\Delta_S = N_{\text{oz}} \cdot \left[\sum_{i=1}^{n} L(t_i) \cdot \Delta t \cdot DF_{\text{EUR}}(t_i) + DF_{\text{EUR}}(t_n) \right] \cdot S_{\text{Gold}} = PV_{\text{Gold}}$$

The result has units of **EUR** (consistent with all other sensitivities) and equals the present value of the gold leg.

Value-at-Risk Methodology

Covariance Matrix

For each day of the time series the risk factors at the payment dates are determined, and log returns are computed from them:

$$r_{DF,i}(t) = \ln \frac{DF(t,t_i)}{DF(t-1,t_i)}, \quad r_{L,i}(t) = \ln \frac{L(t,t_i)}{L(t-1,t_i)}, \quad r_S(t) = \ln \frac{S(t)}{S(t-1)}$$

The covariance matrix $\Sigma \in \mathbb{R}^{(2n+1)\times(2n+1)}$ is estimated as the sample covariance of T returns:

$$\Sigma_{jk} = \frac{1}{T-1} \sum_{t=1}^{T} \left(r_j(t) - \bar{r}_j\right)\left(r_k(t) - \bar{r}_k\right)$$

Portfolio Variance and VaR

The sensitivity vector is:

$$\mathbf{w} = \left(\Delta_{DF(t_1)}, \ldots, \Delta_{DF(t_n)}, \Delta_{L(t_1)}, \ldots, \Delta_{L(t_n)}, \Delta_S\right)^T$$

Portfolio Variance and VaR

$$\sigma_P^2 = \mathbf{w}^T \Sigma \, \mathbf{w}$$

$$\text{VaR}_\alpha(h) = z_\alpha \cdot \sigma_P \cdot \sqrt{h}$$

with $z_{0.95} = 1.645$ and $z_{0.99} = 2.326$, and h = time horizon in days.

Complete Worked Example

The example can be reproduced directly in the calculator: set the number of payments to 2, click **"Load sample data"** in the **Data** tab, all other inputs at their defaults.

Swap Specification

Parameter	Value
EUR Notional	5,000,000 EUR
Gold Notional	1,230 oz (market-consistent, rounded to 10 oz)
EUR Fixed Rate	2.0% p.a.
Gold Spot	4,074.23 EUR/oz (30 Oct 2026)
Term	1 year
Payment Frequency	Semi-annual ($\Delta t = 0.5$)
Number of Payments	2
Pay Leg	EUR

Current Market Values (Last Day of the Sample Data)

From the last row of the sample data (30 Oct 2026). Both payment dates coincide with curve points, so no interpolation is needed.

Quantity	Value	Risk Factor
EUR 6M Zero Rate	2.7609%	$DF(0.5) = e^{-0.027609 \cdot 0.5} = 0.986290$
EUR 1Y Zero Rate	2.9267%	$DF(1.0) = e^{-0.029267 \cdot 1.0} = 0.971157$
Gold Lease 6M	0.4510%	$L(0.5) = 0.004510$
Gold Lease 1Y	0.5033%	$L(1.0) = 0.005033$

Cash Flow Calculation

Worked Example

$V_{\text{Gold}} = 1{,}230 \times 4{,}074.23 = 5{,}011{,}302.90$ EUR

EUR leg (pay):

$$CF_{\text{EUR}}(0.5) = 5{,}000{,}000 \times 0.02 \times 0.5 = 50{,}000.00 \text{ EUR}$$
$$CF_{\text{EUR}}(1.0) = 50{,}000 + 5{,}000{,}000 = 5{,}050{,}000.00 \text{ EUR}$$

Gold leg (receive):

$$CF_{\text{Gold}}(0.5) = 5{,}011{,}302.90 \times 0.004510 \times 0.5 = 11{,}300.49 \text{ EUR}$$
$$\begin{aligned} CF_{\text{Gold}}(1.0) &= 5{,}011{,}302.90 \times 0.005033 \times 0.5 + 5{,}011{,}302.90 \\ &= 12{,}610.94 + 5{,}011{,}302.90 = 5{,}023{,}913.84 \text{ EUR} \end{aligned}$$

Present Value Calculation

Worked Example

$$\begin{aligned} PV_{\text{EUR}} &= 50{,}000 \times 0.986290 + 5{,}050{,}000 \times 0.971157 \\ &= 49{,}314.52 + 4{,}904{,}343.51 = 4{,}953{,}658.03 \text{ EUR} \end{aligned}$$

$$\begin{aligned} PV_{\text{Gold}} &= 11{,}300.49 \times 0.986290 + 5{,}023{,}913.84 \times 0.971157 \\ &= 11{,}145.56 + 4{,}879{,}009.75 = 4{,}890{,}155.32 \text{ EUR} \end{aligned}$$

$$PV_{\text{net}} = 4{,}890{,}155.32 - 4{,}953{,}658.03 = \mathbf{-63{,}502.71} \text{ EUR}$$

Interpretation: The swap has a negative market value. EUR notional and gold value are nearly equal, but the EUR fixed rate (2%) is considerably higher than the lease rates ($\approx 0.5\%$).

Sensitivity Calculation

Worked Example

EUR DF sensitivities $= (CF_{\text{Gold}} - CF_{\text{EUR}}) \times DF$:

$$\begin{aligned} \Delta_{DF(0.5)} &= (11{,}300.49 - 50{,}000) \times 0.986290 = -38{,}168.96 \text{ EUR} \\ \Delta_{DF(1.0)} &= (5{,}023{,}913.84 - 5{,}050{,}000) \times 0.971157 = -25{,}333.76 \text{ EUR} \end{aligned}$$

Lease rate sensitivities $= V_{\text{Gold}} \times \Delta t \times DF \times L$:

$$\begin{aligned} \Delta_{L(0.5)} &= 5{,}011{,}302.90 \times 0.5 \times 0.986290 \times 0.004510 = 11{,}145.56 \text{ EUR} \\ \Delta_{L(1.0)} &= 5{,}011{,}302.90 \times 0.5 \times 0.971157 \times 0.005033 = 12{,}247.21 \text{ EUR} \end{aligned}$$

Gold spot sensitivity $= N_{\text{oz}} \times [\ldots] \times S$:

$$\begin{aligned} \Delta_S &= 1{,}230 \times \big[0.004510 \times 0.5 \times 0.986290 \\ &\quad + 0.005033 \times 0.5 \times 0.971157 + 0.971157\big] \times 4{,}074.23 \\ &= 1{,}230 \times 0.975825 \times 4{,}074.23 = 4{,}890{,}155.32 \text{ EUR} \end{aligned}$$

Sensitivity Summary

Risk Factor	Derivative	Sensitivity (= Deriv. × Factor)
EUR DF(0.5Y)	−38,699.51	−38,168.96 EUR
EUR DF(1.0Y)	−26,086.16	−25,333.76 EUR
Gold Lease(0.5Y)	2,471,299.83	11,145.56 EUR
Gold Lease(1.0Y)	2,433,381.27	12,247.21 EUR
Gold Spot	1,200.26 oz	4,890,155.32 EUR

Covariance Matrix from the Sample Data

From 250 daily log returns (14 Nov 2025 – 30 Oct 2026):

	DF(0.5Y)	DF(1.0Y)	L(0.5Y)	L(1.0Y)	Gold
DF(0.5Y)	7.79×10^{-9}	1.94×10^{-8}	5.61×10^{-8}	8.39×10^{-8}	8.44×10^{-8}
DF(1.0Y)	1.94×10^{-8}	6.16×10^{-8}	-1.19×10^{-7}	-8.89×10^{-8}	3.21×10^{-7}
L(0.5Y)	5.61×10^{-8}	-1.19×10^{-7}	5.13×10^{-4}	4.45×10^{-4}	1.89×10^{-5}
L(1.0Y)	8.39×10^{-8}	-8.89×10^{-8}	4.45×10^{-4}	4.13×10^{-4}	1.84×10^{-5}
Gold	8.44×10^{-8}	3.21×10^{-7}	1.89×10^{-5}	1.84×10^{-5}	1.15×10^{-4}

Volatilities (daily):

Risk Factor	Volatility
EUR DF(0.5Y)	0.0088%
EUR DF(1.0Y)	0.0248%
Gold Lease(0.5Y)	2.2652%
Gold Lease(1.0Y)	2.0323%
Gold Spot	1.0735% ($\approx 17\%$ p.a.)

Value-at-Risk Calculation

Worked Example

Sensitivity vector:

$$\mathbf{w} = (-38{,}169;\ -25{,}334;\ 11{,}146;\ 12{,}247;\ 4{,}890{,}155)^T$$

Portfolio variance:

$$\sigma_P^2 = \mathbf{w}^T \Sigma\, \mathbf{w} = 2{,}760{,}391{,}519 \text{ EUR}^2$$

Portfolio volatility (1 day):

$$\sigma_P = \sqrt{2{,}760{,}391{,}519} = 52{,}539.43 \text{ EUR}$$

Value-at-Risk:

$$\begin{aligned}
\text{VaR}_{95\%,1d} &= 1.645 \times 52{,}539.43 = 86{,}427.36 \text{ EUR} \\
\text{VaR}_{99\%,1d} &= 2.326 \times 52{,}539.43 = 122{,}206.71 \text{ EUR} \\
\text{VaR}_{99\%,10d} &= 122{,}206.71 \times \sqrt{10} = 386{,}451.55 \text{ EUR}
\end{aligned}$$

Interpretation: With the gold spot sensitivity alone the 99% VaR (1 day) would be 122,109 EUR, with the lease rates alone 1,157 EUR and with the EUR discount factors alone 22 EUR. The risk is almost entirely driven by the gold price.

Results Summary

Metric	Value
Swap Valuation	
PV EUR leg	4,953,658.03 EUR
PV Gold leg	4,890,155.32 EUR
Net PV (Pay EUR)	−63,502.71 EUR
Gold Spot Exposure	5,011,302.90 EUR
Risk Metrics	
Portfolio volatility (1d)	52,539.43 EUR
VaR 95% (1 day)	86,427.36 EUR
VaR 99% (1 day)	122,206.71 EUR
VaR 99% (10 days)	386,451.55 EUR
Main Drivers	
Largest sensitivity	Gold Spot (4,890,155 EUR)
Largest volatility	Gold Lease 6M (2.27%/day)

Default case: With the preset 6 semi-annual payments (3 years) the sample data give a net PV of −198,254.58 EUR, a 95% VaR (1 day) of 82,386.98 EUR and a 99% VaR (1 day) of 116,493.69 EUR.

Using the Calculator

The calculator has four tabs: **Swap Details**, **Market Data (DFs)**, **Data** and **Risk Metrics**.

Recommended Workflow

1. **Load data** (tab "Data")
   "Load sample data" or upload your own CSV file.
   ⇒ EUR DFs, lease rates and the covariance matrix are calculated; the gold price (last row) and a market-consistent gold notional are applied to the swap details. A green message confirms the import.

2. **Check swap details** (tab "Swap Details")
   EUR notional, gold notional, fixed rate, gold spot, number of payments, frequency, pay leg.

3. **Click "Calculate Swap"**
   ⇒ PV EUR leg, PV gold leg, gold spot exposure and net PV are computed.

4. **Click "Calculate Risk"** (tab "Risk Metrics")
   Before clicking, optionally adjust the **time horizon** h **(days)** (default: 1 day).
   ⇒ Sensitivities, covariance matrix, VaR 95% and 99% (scaled by $\sqrt{h}$) are shown.

Tab "Data"

Sample Data Gold/EUR (251 days): The calculator contains realistic synthetic data from 14 Nov 2025 to 30 Oct 2026:

- EUR swap curve (zero-coupon, continuous) as in the swap and CCS calculators, tenors 3M, 6M, 1Y, 2Y, 3Y, 5Y, 7Y, 10Y
- Gold lease curve (zero-coupon, continuous) with the same tenors
- Gold price in EUR per troy ounce

"Load sample data" applies the data, **"As CSV"** saves them as a template for your own data in the file `Gold_Sample_Data_EUR_251.csv`.

Upload Historical CSV: Your own time series in one of the formats in Section 8. The file must be stored locally on your computer; it is processed only in your browser and is not uploaded.

Synthetic data: "Generate Synthetic Data" creates correlated time series starting from the current market values (Cholesky decomposition; volatility 1% p.a., gold price 15% p.a.) and replaces only the covariance matrix. The random number generator uses a fixed seed: the same inputs always give the same series and the same VaR. The data are for illustration only.

Export: "Export Time Series" saves the risk factors at the payment dates (Section 8.3).

Important Notes

Important Note

- The **gold spot** is always taken from the input field. When data are loaded it is set to the value of the last row, but it can then be changed for a hypothetical valuation.
- The **covariance matrix** comes from the loaded or synthetic data. Without data, a default matrix is used.
- **Log returns** are used for all risk factors (EUR DFs, lease rates, gold price): $r_t = \ln(x_t/x_{t-1})$.
- If the **number of payments or the frequency** is changed, the calculator recalculates DFs, lease rates and covariance matrix from the loaded yield curves; synthetic data are discarded. With n payments the matrix has dimension $(2n+1) \times (2n+1)$.
- Manual changes in the "Market Data" tab apply until the number of payments or the frequency is changed.

CSV Data Formats

Format 1 – Yield Curves (Recommended)

```
Datum,EUR_0.25Y,...,EUR_10Y,LEASE_0.25Y,...,LEASE_10Y,GOLD_SPOT
2025-11-14,1.9710,...,2.6712,0.4200,...,0.8000,3480.00
...
2026-10-30,2.5916,...,3.2451,0.4171,...,0.7421,4074.23
```

- Columns are recognised from the header: `EUR_<tenor>`, `LEASE_<tenor>` and a column starting with `GOLD`
- Tenors with `M` (months) or `Y` (years); number and position are free, values are interpolated. The old format `EUR_1M,...,EUR_10Y,LEASE_1M,...,LEASE_10Y,GOLD_SPOT` is recognised as well
- Zero rates and lease rates in **percent** (continuous), lease rates above 0
- Gold spot in EUR/oz
- One trading day per row, in ascending order; the last row provides the current values. At least 30, recommended 251 rows
- Separator comma or semicolon; with semicolon a decimal comma is allowed. Encoding UTF-8

Format 2 – Risk Factors at the Payment Dates

```
Date,EUR_DF_0.50Y,EUR_DF_1.00Y,LEASE_0.50Y,LEASE_1.00Y,GOLD_SPOT
2025-11-14,0.98995083,0.98036238,0.00460000,0.00520000,3480.0000
```

- Optional date or day column, then n EUR discount factors, n lease rates (decimal) and the gold spot
- The number of columns must match the payment schedule ($2n + 1$ data columns)
- All values must be positive (log returns); at least 2 rows

Time Series Export

The export uses format 2 with a day column:

```
Day,EUR_DF_0.50Y,EUR_DF_1.00Y,LEASE_0.50Y,LEASE_1.00Y,GOLD_SPOT
1,0.98995083,0.98036238,0.00460000,0.00520000,3480.0000
```

The exported file can be uploaded again directly.

Glossary

Term	Meaning
Discount Factor	Discounting factor: $DF(t) = e^{-r \cdot t}$
Gold Lease Rate	Interest rate for borrowing physical gold
VaR	Value-at-Risk: loss that is not exceeded with a given probability
Sensitivity	Elasticity: derivative times risk factor ($\partial PV / \partial x \cdot x$)
Log return	$\ln(x_t / x_{t-1})$: dimensionless return of a risk factor
Zero Rate	Continuous interest rate with no coupon effect
Tenor	Maturity
Notional	Principal amount of a derivative contract
Pay Leg	The side of the swap that the holder pays
Time horizon	Number of days h; the VaR is scaled by $\sqrt{h}$

Formula Reference

Quantity	Formula
Discount Factor	$DF(t) = e^{-r(t) \cdot t}$
EUR Leg PV	$\sum_i CF_{\text{EUR}}(t_i) \cdot DF(t_i)$
Gold Leg PV	$\sum_i CF_{\text{Gold}}(t_i) \cdot DF(t_i)$
EUR DF Sensitivity	$[CF_{\text{Gold}}(t_i) - CF_{\text{EUR}}(t_i)] \cdot DF(t_i)$
Lease Sensitivity	$V_{\text{Gold}} \cdot \Delta t \cdot DF(t_i) \cdot L(t_i)$
Gold Spot Sensitivity	$N_{\text{oz}} \cdot [\sum_i L_i \cdot \Delta t \cdot DF_i + DF_n] \cdot S$
Log return	$r_t = \ln(x_t / x_{t-1})$
Portfolio Variance	$\mathbf{w}^T \Sigma \, \mathbf{w}$
VaR	$z_\alpha \cdot \sqrt{\mathbf{w}^T \Sigma \, \mathbf{w}} \cdot \sqrt{h}$