Portfolio VaR Calculator 2

Mathematical and Technical Documentation

Interest-Rate-Sensitive Products in Four Currency Areas: EUR, USD, GBP, CHF

English

Wolfgang Boehme

wolfgang.boehme@economic-modeling.de

www.economic-modeling.de

25 October 2026

The documentation on this website was developed with the support of the AI Claude (Anthropic).

0 Table of Contents

1 Overview and Objective **2**
1.1 Comparison: Naive vs. Correct . . . 2

2 Data and Risk Factors **2**
2.1 Market Data CSV . . . 2
2.2 Linear Interpolation of the Yield Curve . . . 3

3 Discount Factors and Log-Returns **3**
3.1 Spread-Adjusted Discount Factor . . . 3
3.2 Log-Returns as a Uniform Risk Variable . . . 3

4 The Global Covariance Matrix **4**
4.1 Construction . . . 4
4.2 Block Structure . . . 4

5 Instrument Valuation **4**
5.1 Fixed-Income Bond . . . 4
5.2 Fixed-Rate Swap . . . 5
5.3 Cross-Currency Swap (CCS) . . . 5
5.4 FX Forward . . . 5

6 Sensitivities **6**
6.1 Sensitivity to Discount Factors . . . 6
6.2 Sensitivity for FX Forwards . . . 6

7 VaR Calculation **6**
7.1 Mapping to Risk Factors . . . 6
7.2 Portfolio Variance . . . 6
7.3 VaR Formula . . . 6
7.4 Individual VaR vs. Portfolio VaR . . . 7

8 Technical Implementation **7**
8.1 Data Structure: `r2State` . . . 7
8.2 Calculation Workflow . . . 8
8.3 Factor List (Order in the Matrix) . . . 8

9 Transparency Tab: Matrices and Cash Flows **9**

10 Limitations and Outlook **9**
10.1 Known Simplifications . . . 9
10.2 Possible Extensions . . . 9

1 Overview and Objective

Portfolio VaR Calculator 2 computes the **Value at Risk (VaR)** of a portfolio of interest-rate-sensitive products using a single, historically estimated covariance matrix. Unlike approaches that consider each risk factor in isolation, this calculator captures *correlations among all risk factors simultaneously* — both within a currency area and across different currency areas.

1.1 Comparison: Naive vs. Correct

	Naive Summation	**Calculator 2**
Assumption	$\rho_{ij} = 1$ for all i, j	historically measured
Aggregation	$\mathrm{VaR}_{\text{total}} = \sum_k \mathrm{VaR}_k$	$\mathrm{VaR}_{\text{total}} = z_\alpha \sqrt{\mathbf{w}^\top \boldsymbol{\Sigma}\, \mathbf{w}}$
Risk factors	per product separately	shared 29×29 matrix
Sub-additivity	violated	guaranteed
Diversification	ignored	explicitly reported

Sub-additivity (Artzner et al., 1999)

A risk measure ρ is called *sub-additive* if for arbitrary portfolios A and B:

$$\rho(A + B) \;\leq\; \rho(A) + \rho(B).$$

The covariance-based VaR is sub-additive because the triangle inequality holds for the Euclidean norm:

$$\sqrt{\mathbf{w}^\top \boldsymbol{\Sigma}\, \mathbf{w}} \;\leq\; \sqrt{\mathbf{w}_A^\top \boldsymbol{\Sigma}_{AA}\, \mathbf{w}_A} + \sqrt{\mathbf{w}_B^\top \boldsymbol{\Sigma}_{BB}\, \mathbf{w}_B}.$$

2 Data and Risk Factors

2.1 Market Data CSV

The calculator reads historical end-of-day prices from a CSV file with $T = 252$ rows (trading days) and 104 columns. For Calculator 2, 29 columns are used, as listed in the table below.

Block	**Content**	**Count**	**Tenors (years)**
EUR Zero Rates	$r^{\mathrm{EUR}}(t_k)$	5	0.25, 1, 2, 5, 10
USD Zero Rates	$r^{\mathrm{USD}}(t_k)$	5	0.25, 1, 2, 5, 10
GBP Zero Rates	$r^{\mathrm{GBP}}(t_k)$	5	0.25, 1, 2, 5, 10
CHF Zero Rates	$r^{\mathrm{CHF}}(t_k)$	5	0.25, 1, 2, 5, 10
FX Spots	EUR/USD, EUR/GBP, EUR/CHF	3	—
Credit Spread A	$s^{\mathrm{A}}(t_k)$	3	1, 5, 10
Credit Spread BBB	$s^{\mathrm{BBB}}(t_k)$	3	1, 5, 10
Total		**29**	

All interest rates are stored in the CSV as percentages (e.g. 3.50) and are converted internally to decimal form (0.0350) by dividing by 100.

2.2 Linear Interpolation of the Yield Curve

For a cash flow date t lying between two nodes $t_k < t < t_{k+1}$, the corresponding zero rate is linearly interpolated:

$$r(t) = r(t_k) \cdot \frac{t_{k+1} - t}{t_{k+1} - t_k} + r(t_{k+1}) \cdot \frac{t - t_k}{t_{k+1} - t_k}.$$

For dates outside the range of nodes, the nearest boundary value is used (constant extrapolation).

3 Discount Factors and Log-Returns

3.1 Spread-Adjusted Discount Factor

For an instrument with a credit rating (A or BBB), the credit spread $s(t)$ is added directly to the zero rate. The discount factor for maturity t is:

$$\mathrm{DF}(t,\, d) = \exp\!\big[-\big(r(t,d) + s(t,d)\big) \cdot t\big],$$

where $d \in \{1, \ldots, T\}$ denotes the trading day. Without a credit spread, $s \equiv 0$.

3.2 Log-Returns as a Uniform Risk Variable

The central methodological decision of the calculator is the use of **log-returns of discount factors** as risk factors. This ensures that all entries of the covariance matrix are expressed in the same unit — in contrast to mixing interest rate differences (bp) with price returns.

The log-return of the discount factor for tenor t_k and currency c from day $d-1$ to day d is:

$$\begin{aligned} \ell_k^c(d) &= \ln\!\left(\frac{\mathrm{DF}(t_k,\, d)}{\mathrm{DF}(t_k,\, d-1)}\right) = \ln\!\left(\frac{e^{-(r_d^c(t_k) + s_d(t_k))\, t_k}}{e^{-(r_{d-1}^c(t_k) + s_{d-1}(t_k))\, t_k}}\right) \\ &= -t_k \cdot \big[(r_d^c(t_k) + s_d(t_k)) - (r_{d-1}^c(t_k) + s_{d-1}(t_k))\big]. \end{aligned} \tag{1}$$

This equals the negative, maturity-weighted change in the spread-adjusted interest rate. The credit spread is therefore *automatically* integrated into the log-return — no separate spread risk block is required.

For FX spots S_d (quoted as EUR/CCY), the log-return is likewise used:

$$\ell^{\mathrm{FX}}(d) = \ln\!\left(\frac{S_d}{S_{d-1}}\right).$$

From $T = 252$ daily observations, $T - 1 = 251$ log-return observations are obtained per risk factor.

4 The Global Covariance Matrix

4.1 Construction

Let $\ell_1(d), \ldots, \ell_{29}(d)$ be the log-returns of all 29 risk factors on day d, with means $\bar{\ell}_i = \frac{1}{T-1}\sum_{d=1}^{T-1} \ell_i(d)$.

The global covariance matrix $\boldsymbol{\Sigma} \in \mathbb{R}^{29\times 29}$ is estimated from the historical log-returns:

$$\Sigma_{ij} = \frac{1}{T-2}\sum_{d=1}^{T-1}\big(\ell_i(d) - \bar{\ell}_i\big)\big(\ell_j(d) - \bar{\ell}_j\big), \quad i,j \in \{1,\ldots,29\}.$$

The matrix is symmetric ($\Sigma_{ij} = \Sigma_{ji}$) and positive semi-definite. With $T-1 = 251$ observations and $N = 29$ factors, the ratio $251/29 \approx 8.7$ is sufficient for a stable covariance estimate (rule of thumb: > 5).

4.2 Block Structure

The matrix decomposes naturally into blocks:

$$\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}^{5\times 5}_{\text{EUR}} & \boldsymbol{\Sigma}_{\text{EUR,USD}} & \cdots & \boldsymbol{\sigma}_{\text{EUR,FX}} & \boldsymbol{\Sigma}_{\text{EUR,CS}} \\ \boldsymbol{\Sigma}_{\text{USD,EUR}} & \boldsymbol{\Sigma}^{5\times 5}_{\text{USD}} & \cdots & \vdots & \vdots \\ \vdots & & \ddots & & \vdots \\ \boldsymbol{\sigma}^{\top}_{\text{FX,EUR}} & \cdots & \cdots & \boldsymbol{\Sigma}^{3\times 3}_{\text{FX}} & \boldsymbol{\Sigma}_{\text{FX,CS}} \\ \boldsymbol{\Sigma}_{\text{CS,EUR}} & \cdots & \cdots & \boldsymbol{\Sigma}_{\text{CS,FX}} & \boldsymbol{\Sigma}^{6\times 6}_{\text{CS}} \end{pmatrix}.$$

The diagonal blocks $\boldsymbol{\Sigma}_{kk}$ contain the variances and internal covariances of each currency area or product class. The off-diagonal blocks $\boldsymbol{\Sigma}_{kl}$ ($k \neq l$) capture the *cross-correlations* between different risk factors — for example, the historical correlation between EUR rates and USD rates.

5 Instrument Valuation

5.1 Fixed-Income Bond

A bond with notional N, coupon rate c p.a., term T_{Bond} and payment frequency f (1 = annual, 2 = semi-annual) generates cash flows at times $t_j = j/f$, $j = 1, \ldots, n = f \cdot T_{\text{Bond}}$:

$$\text{CF}_j = \frac{N \cdot c}{f} + N \cdot \mathbf{1}_{[j=n]},$$

where the principal N is repaid only at the final date $t_n = T_{\text{Bond}}$.

The present value in EUR is:

$$\text{PV}_{\text{Bond}} = \frac{1}{S_{\text{CCY/EUR}}}\sum_{j=1}^{n} \text{CF}_j \cdot \text{DF}(t_j),$$

where $S_{\text{CCY/EUR}}$ is the FX spot (units of foreign currency per EUR), i.e. EUR/USD for a USD bond.

5.2 Fixed-Rate Swap

In a fixed-rate swap, a fixed coupon is exchanged for a floating rate. Since the floating leg is valued at par, its present value is:

$$\text{PV}_{\text{float}} = N \cdot (1 - \text{DF}(T_{\text{Swap}})).$$

The total present value of a **payer swap** (pay fixed, receive floating) in EUR is:

$$\text{PV}_{\text{Swap}}^{\text{Payer}} = \frac{1}{S_{\text{CCY/EUR}}} \left[N \cdot (1 - \text{DF}(T)) - \sum_{j=1}^{n} \frac{N \cdot c}{f} \cdot \text{DF}(t_j) - N \cdot \text{DF}(T) \right].$$

For the **receiver swap**, all signs are reversed.

5.3 Cross-Currency Swap (CCS)

A CCS exchanges cash flows in currency 1 for cash flows in currency 2. At inception the notional amounts are chosen to match the current FX spot:

$$N_2 = N_1 \cdot S_{12},$$

where S_{12} is the spot rate (units of CCY2 per CCY1).

Each leg is valued like a bond (with principal repayment at maturity):

$$\text{PV}_{\text{Leg1}} = \frac{1}{S_{\text{CCY1/EUR}}} \sum_j \text{CF}_{1,j} \cdot \text{DF}_1(t_j),$$

$$\text{PV}_{\text{Leg2}} = \frac{1}{S_{\text{CCY2/EUR}}} \sum_j \text{CF}_{2,j} \cdot \text{DF}_2(t_j).$$

The net present value (from the perspective of the payer of leg 1) is:

$$\text{PV}_{\text{CCS}} = -\text{PV}_{\text{Leg1}} + \text{PV}_{\text{Leg2}}.$$

5.4 FX Forward

An FX forward (purchase of CCY1 against CCY2 at forward rate F, maturity T) has the following present value in EUR:

$$\text{PV}_{\text{FX}} = \frac{N \cdot (F - S_0) \cdot \text{DF}_{\text{CCY1}}(T)}{S_{\text{CCY2/EUR}}},$$

where S_0 is the current spot rate and $\text{DF}_{\text{CCY1}}(T)$ is the discount factor in the base currency.

6 Sensitivities

6.1 Sensitivity to Discount Factors

The VaR calculation requires, for each position j, the sensitivity of the present value to the discount factor, i.e. the partial derivative $\delta_j = \partial \mathrm{PV} / \partial \mathrm{DF}(t_j)$:

$$\delta_j = \frac{\partial \mathrm{PV}}{\partial \mathrm{DF}(t_j)} = \frac{\mathrm{CF}_j}{S_{\mathrm{CCY/EUR}}}.$$

For a payer swap, $\mathrm{CF}_j < 0$ (fixed payment), hence $\delta_j < 0$. This is intuitively correct: a rise in rates (falling DFs) increases the value of a fixed payment obligation for the receiver and hurts the payer.

6.2 Sensitivity for FX Forwards

For an FX forward, the primary risk factor is the FX spot S_0. The sensitivity to a log-return of the spot is:

$$\delta^{\mathrm{FX}} \approx \frac{\partial \mathrm{PV}}{\partial \ln S_0} = \frac{N \cdot S_0}{S_{\mathrm{CCY2/EUR}}}.$$

7 VaR Calculation

7.1 Mapping to Risk Factors

Each position j is mapped to the nearest node t_k in the global factor list:

$$k^*(j) = \arg\min_k |t_k - t_j| ,$$

where bonds with a credit rating are mapped to the corresponding credit spread factors (CS_A or CS_BBB) — not to the risk-free yield curve. This ensures that the historical correlation structure of the spread-adjusted discount factors is correctly captured.

7.2 Portfolio Variance

Let $\mathbf{w} \in \mathbb{R}^{29}$ be the sensitivity vector, where $w_k = \sum_{j:\, k^*(j)=k} \delta_j$ is the aggregated sensitivity to risk factor k. The portfolio variance is:

$$\sigma_P^2 = \mathbf{w}^\top \boldsymbol{\Sigma}\, \mathbf{w} = \sum_{i=1}^{29} \sum_{j=1}^{29} w_i\, \Sigma_{ij}\, w_j.$$

7.3 VaR Formula

Under the normality assumption for log-returns:

$$\boxed{\mathrm{VaR}_\alpha = z_\alpha \cdot \sqrt{\mathbf{w}^\top \boldsymbol{\Sigma}\, \mathbf{w}}}$$

where $z_\alpha = \Phi^{-1}(\alpha)$ is the α-quantile of the standard normal distribution. For $\alpha = 99\,\%$, $z_{0.99} \approx 2.326$.

7.4 Individual VaR vs. Portfolio VaR

The calculator computes **both** quantities from the *same* covariance matrix:

Quantity	Formula
Individual VaR instrument i	$\mathrm{VaR}_i = z_\alpha \sqrt{\mathbf{w}_i^\top \boldsymbol{\Sigma}_{kk}\, \mathbf{w}_i}$
Tab total VaR (with corr.)	$\mathrm{VaR}_{\mathrm{Tab}} = z_\alpha \sqrt{\mathbf{w}_{\mathrm{Tab}}^\top \boldsymbol{\Sigma}\, \mathbf{w}_{\mathrm{Tab}}}$
Portfolio VaR (all tabs)	$\mathrm{VaR}_P = z_\alpha \sqrt{\mathbf{w}^\top \boldsymbol{\Sigma}\, \mathbf{w}}$
Undiversified VaR	$\mathrm{VaR}_{\mathrm{undiv}} = \sum_k \mathrm{VaR}_k$
Diversification benefit	$\Delta\mathrm{VaR} = \mathrm{VaR}_{\mathrm{undiv}} - \mathrm{VaR}_P \geq 0$

Sub-additivity $\mathrm{VaR}_P \leq \mathrm{VaR}_{\mathrm{undiv}}$ is *guaranteed*, since all VaRs are computed from the same positive semi-definite matrix and the Cauchy–Schwarz inequality holds.

Example: EUR bond (Rating A) + USD bond (Rating BBB)

Inputs:
Bond 1: EUR, 100,000 €, coupon 5%, 4 years, rating A
Bond 2: USD, 80,000 USD, coupon 4.5%, 3 years, rating BBB
EUR/USD = 1.11

Risk factors:
Bond 1 → CS_A (1Y, 5Y, 10Y), nearest tenor: 1Y and 5Y
Bond 2 → CS_BBB (1Y, 5Y), converted to EUR via 1/1.11

Covariance structure:
$\mathrm{Cov}(\ell^{\mathrm{CS_A}}, \ell^{\mathrm{CS_BBB}}) < \mathrm{Cov}(\ell^{\mathrm{CS_A}}, \ell^{\mathrm{CS_A}}) \Rightarrow$ Portfolio VaR < sum of individual VaRs

Typical result:
VaR Bond 1: €2,800 VaR Bond 2: €1,600
Sum: €4,400 Portfolio VaR: €3,700 Saving: €700 (15%)

8 Technical Implementation

8.1 Data Structure: `r2State`

All calculation results are stored in a global JavaScript object `r2State`:

```
r2State = {
  csvLoaded:  bool,          // CSV successfully loaded?
  rawData:    {col: [val]},  // Raw data per column
```

```
  logReturns: {col: [lr]},   // Log-returns per risk factor
  fxSpots:    {pair: val},   // Current FX spots
  globalCov:  [[...]],       // 29x29 covariance matrix
  positions:  [{...}],       // All active positions
  products: {
    bonds:  {bonds, positions},
    swaps:  {swaps, positions},
    ccs:    {ccsList, positions},
    fx:     {fxList, positions}
  }
}
```

8.2 Calculation Workflow

Step 1. Load CSV: All 29 required columns are read and validated. Missing columns trigger an error message.

Step 2. Compute log-returns: For each of the 29 risk factors, 251 log-returns are computed from the 252 daily observations. For rate factors: DF-based (see above). For FX: directly from the spot.

Step 3. Build covariance matrix: $\mathbf{\Sigma}$ is computed as a 29×29 matrix using the historical estimator. Complexity: $\mathcal{O}(N^2 \cdot T) = \mathcal{O}(29^2 \cdot 251)$.

Step 4. Value the product: Cash flows, discount factors, present value and sensitivities are computed per instrument.

Step 5. Risk factor mapping: Each position is assigned to the nearest node in `R2_ALL_FACTORS`. Rated bonds $\rightarrow$ CS factors; swaps/CCS $\rightarrow$ currency zero curve; FX $\rightarrow$ FX spot factor.

Step 6. Compute VaR: $\sigma_P^2 = \mathbf{w}^\top \mathbf{\Sigma} \mathbf{w}$, $\text{VaR} = z_\alpha \cdot \sigma_P$.

Step 7. Diversification: Individual VaRs from diagonal blocks of the *same* matrix; their sum $\geq$ portfolio VaR.

8.3 Factor List (Order in the Matrix)

Index	Factor	Block	Tenor
0–4	EUR_Zero_3M/1Y/2Y/5Y/10Y	EUR Zero	0.25, 1, 2, 5, 10 yr
5–9	USD_Zero_3M/1Y/2Y/5Y/10Y	USD Zero	0.25, 1, 2, 5, 10 yr
10–14	GBP_Zero_3M/1Y/2Y/5Y/10Y	GBP Zero	0.25, 1, 2, 5, 10 yr
15–19	CHF_Zero_3M/1Y/2Y/5Y/10Y	CHF Zero	0.25, 1, 2, 5, 10 yr
20	EUR/USD	FX Spot	—
21	EUR/GBP	FX Spot	—
22	EUR/CHF	FX Spot	—
23–25	CreditSpread_A_1Y/5Y/10Y	CS A	1, 5, 10 yr
26–28	CreditSpread_BBB_1Y/5Y/10Y	CS BBB	1, 5, 10 yr

9 Transparency Tab: Matrices and Cash Flows

The calculator provides three levels of information in the *Matrices & Cash Flows* tab:

1. **Local covariance matrices** — one 5×5 block (rate blocks) or 3×3 block (FX, CS) each, displayed as a heat map. Colour intensity encodes the magnitude of the covariance relative to the diagonal variance (approximately the correlation).
2. **Global covariance matrix** — the full 29×29 matrix, of which the first 20 factors are displayed. The complete matrix can be exported as a CSV (values in exponential notation).
3. **Cash flow table** — for each active position, the payment date t_j, cash flow CF, present value PV (EUR), sensitivity δ_j and the assigned risk factor are listed. Export to CSV is available.

10 Limitations and Outlook

10.1 Known Simplifications

1. **Normality assumption:** The formula $\text{VaR} = z_\alpha \sigma_P$ is exact only if log-returns are normally distributed. In practice, financial returns are leptokurtic (fat tails). Historical simulation or extreme value theory can correct this.
2. **Single-factor mapping:** Each position is assigned to the *nearest* tenor. A more precise method would be linear interpolation between two adjacent factors (rho-mapping).
3. **Swap floating leg:** The present value of the floating leg is approximated by $N \cdot (1 - \text{DF}(T))$. This is exact for LIBOR swaps without spread; OIS-discounted swaps would require separate discounting and forward curves.
4. **FX forward:** The sensitivity $\delta^{\text{FX}} \approx N \cdot S_0 / S_{\text{CCY2/EUR}}$ is a delta approximation. For exotic options or non-linear products, a more complete gamma treatment would be required.
5. **Covariance stationarity:** The historical estimate over 251 days implicitly assumes that the covariance structure is stationary. In crisis periods, correlations can vary significantly (correlation breakdown). GARCH models or exponential weighting (EWMA) could address this.

10.2 Possible Extensions

- Historical simulation (full revaluation on each day)
- Stressed VaR (crisis windows 2008–2009, 2020)
- Expected shortfall (CVaR) as a more coherent risk measure
- EWMA covariance matrix with decay factor $\lambda = 0.94$ (RiskMetrics)
- Gamma terms for interest rate options and swaptions

10 Summary of Key Formulae

Quantity	**Formula**
Discount factor	$\mathrm{DF}(t,d) = e^{-(r_d(t)+s_d(t))\,t}$
Log-return DF	$\ell_k(d) = \ln(\mathrm{DF}(t_k,d)/\mathrm{DF}(t_k,d-1))$
Covariance estimate	$\Sigma_{ij} = \frac{1}{T-2}\sum_d(\ell_i(d)-\bar{\ell}_i)(\ell_j(d)-\bar{\ell}_j)$
Sensitivity bond/swap	$\delta_j = \mathrm{CF}_j/S_{\mathrm{CCY/EUR}}$
Portfolio variance	$\sigma_P^2 = \mathbf{w}^\top \boldsymbol{\Sigma}\,\mathbf{w}$
VaR (normal distribution)	$\mathrm{VaR}_\alpha = z_\alpha \cdot \sigma_P = z_\alpha\sqrt{\mathbf{w}^\top \boldsymbol{\Sigma}\,\mathbf{w}}$
Sub-additivity	$\mathrm{VaR}_P \le \sum_k \mathrm{VaR}_k$
Diversification benefit	$\Delta\mathrm{VaR} = \sum_k \mathrm{VaR}_k - \mathrm{VaR}_P \ge 0$

Calculator: `rechner2.html` Data: `market_data_251days.csv` 252 trading days, 29 risk factors, 29×29 covariance matrix