Exercise Collection

Portfolio VaR Calculator 2

Interest-Rate-Sensitive Products in Four Currency Areas

English

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All numerical exercises can be checked with the calculator `rechner2.html` and the data set `market_data_251days.csv`.

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1 Block A: Discount Factors and Log-Returns

Exercise 1 — *E — Discount Factor*

The EUR zero rate for a maturity of $t = 5$ years is given as

$$r^{\mathrm{EUR}}(5) = 3.50\,\%.$$

(a) Calculate the discount factor $\mathrm{DF}(5)$ without a credit spread.

(b) A EUR bond has rating A. The 5-year credit spread is $s^{\mathrm{A}}(5) = 0.80\,\%$. Calculate the spread-adjusted discount factor $\mathrm{DF}^{\mathrm{A}}(5)$.

(c) By what percentage is $\mathrm{DF}^{\mathrm{A}}(5)$ lower than $\mathrm{DF}(5)$? What does this mean economically?

Exercise 2 — *E — Log-Return of the Discount Factor*

On two consecutive trading days the following EUR zero rates are observed for $t = 2$ years:

$$r^{\mathrm{EUR}}_{\mathrm{Mon}}(2) = 3.20\,\%, \qquad r^{\mathrm{EUR}}_{\mathrm{Tue}}(2) = 3.35\,\%.$$

(a) Calculate $\mathrm{DF}_{\mathrm{Mon}}(2)$ and $\mathrm{DF}_{\mathrm{Tue}}(2)$.

(b) Calculate the log-return $\ell = \ln(\mathrm{DF}_{\mathrm{Tue}}(2)/\mathrm{DF}_{\mathrm{Mon}}(2))$.

(c) Show numerically that $\ell \approx -t \cdot \Delta r = -2 \cdot 0.0015$. When is this approximation exact?

(d) What is the sign of ℓ when interest rates rise? What does this mean for a bond buyer?

Exercise 3 — *M — Linear Interpolation of the Yield Curve*

The EUR zero curve has the following nodes:

Tenor	1Y	2Y	5Y	10Y
Rate (%)	3.20	3.40	3.70	4.00

(a) Use linear interpolation to calculate the zero rates for $t = 3$ years and $t = 4$ years.

(b) Calculate DF(3) and DF(4) using the interpolated rates.

(c) A bond pays a coupon of 10,000 EUR at $t = 3.5$ years. Which rate do you use for discounting? Calculate the present value of this single cash flow.

(d) What happens at $t = 0.5$ years? The calculator uses constant extrapolation — explain why this is a reasonable simplification.

2 Block B: Valuation of the Instruments

Exercise 4 — ***E — Bond Valuation***

Value the following EUR bond (without credit spread):

Notional	100,000 EUR
Coupon	4.00 % p. a., annual
Maturity	3 years

EUR zero rates: $r(1) = 3.20\,\%$, $r(2) = 3.40\,\%$, $r(3) = 3.55\,\%$.

(a) List all cash flows with their payment dates.

(b) Calculate the present value of each cash flow.

(c) Determine the total present value $\mathrm{PV}_{\mathrm{Bond}}$.

(d) How does $\mathrm{PV}_{\mathrm{Bond}}$ change if all zero rates rise by 50 bp? Calculate the change and state the duration (DV01) in EUR/bp.

(e) Check your result with the calculator.

Exercise 5 — ***M — USD Bond with Credit Spread***

A USD bond with rating BBB is to be valued in EUR.

Notional	200,000 USD
Coupon	5.50 % p. a., semi-annual
Maturity	5 years
EUR/USD spot	1.11

USD zero rates: $r^{\mathrm{USD}}(0.5) = 4.80\,\%$, $r^{\mathrm{USD}}(1) = 4.90\,\%$, $r^{\mathrm{USD}}(2) = 5.00\,\%$, $r^{\mathrm{USD}}(5) = 5.20\,\%$.
BBB credit spread: $s(1) = 1.20\,\%$, $s(5) = 1.80\,\%$.

(a) How many cash flows does this bond have?

(b) To which of the 29 risk factors in the calculator is this bond sensitive? Justify your choice.

(c) Calculate the present value of the first coupon cash flow (at $t = 0.5$ years) in EUR.

(d) Why is the credit spread *added* to the USD zero rate rather than treated separately? What advantage does this offer for the covariance matrix?

Exercise 6 — *M — Fixed-Rate Swap*

Analyse a EUR payer swap:

Notional	500,000 EUR
Fixed rate	3.00 % p. a., annual
Maturity	5 years

Current EUR zero rates: $r(1) = 3.20\,\%$, $r(2) = 3.40\,\%$, $r(3) = 3.55\,\%$, $r(5) = 3.70\,\%$.

(a) The calculator approximates the present value of the floating leg by $\mathrm{PV}_{\text{float}} = N \cdot (1 - \mathrm{DF}(T))$. Explain why this formula holds. (*Hint: replication argument*)

(b) Calculate $\mathrm{PV}_{\text{float}}$.

(c) Calculate $\mathrm{PV}_{\text{fixed}}$ (present value of the fixed leg).

(d) Calculate $\mathrm{PV}_{\text{Swap}} = \mathrm{PV}_{\text{float}} - \mathrm{PV}_{\text{fixed}}$. Is the swap in the money or out of the money? Justify your answer with reference to market rates.

(e) What would $\mathrm{PV}_{\text{Swap}}$ be if the swap were entered into *exactly at the current market rate* (at-the-money)?

Exercise 7 — *M — Cross-Currency Swap*

A EUR/USD CCS is entered into:

Leg 1 (pay)	EUR, 1,000,000 EUR, 3.00 % p. a.
Leg 2 (receive)	USD, rate 4.50 % p. a.
Maturity	3 years, annual
EUR/USD spot	1.11

(a) What is the USD notional? Why is it chosen this way?

(b) To which risk factors is this CCS sensitive? List all relevant factors from the list of 29 factors.

(c) The calculator maps leg 1 to EUR risk factors and leg 2 to USD risk factors. Why can the off-diagonal covariance $\Sigma_{\text{EUR,USD}}$ have a considerable effect on the VaR of this CCS?

(d) Sketch the situation in which $\text{PV}_{\text{CCS}} > 0$ (from the perspective of the EUR payer).

Exercise 8 *E — FX Forward*

A company buys GBP and sells EUR forward:

Notional	300,000 GBP
Forward rate	EUR/GBP $= 0.870$
Spot rate	EUR/GBP $= 0.878$
Maturity	1 year

GBP zero rate: $r^{\text{GBP}}(1) = 5.10\,\%$.

(a) Calculate PV_{FX} in EUR.

(b) The calculator's sensitivity is $\delta^{\text{FX}} \approx N \cdot S_0 / S_{\text{CCY2/EUR}}$. Calculate δ^{FX} for this transaction.

(c) To which of the three FX factors (EUR/USD, EUR/GBP, EUR/CHF) is this forward mapped?

(d) Explain intuitively: why does PV_{FX} rise when GBP appreciates against EUR (EUR/GBP falls)?

3 Block C: Covariance Matrix and Risk Factors

Exercise 9 *E — Structure of the Covariance Matrix*

The calculator estimates a 29×29 covariance matrix from 251 log-return observations.

(a) How many independent parameters does a symmetric 29×29 matrix contain? (*Hint: diagonal + upper triangle*)

(b) The ratio of observations to factors is $251/29 \approx 8.7$. Why is a higher ratio better for a stable estimate? What happens if the ratio is < 1?

(c) Name three pairs of risk factors for which you expect a *high* positive correlation, and give reasons.

(d) Name one pair with an expected *negative* correlation and give an economic reason.

Exercise 10 — *M — Covariance and Correlation*

The following entries of the covariance matrix are read from the calculator (tab *Matrices & Cash Flows*) (values $\times 10^{-8}$):

Factors i, j	$\Sigma_{ij} \times 10^8$
EUR_Zero_2Y / EUR_Zero_2Y (variance)	420
USD_Zero_2Y / USD_Zero_2Y (variance)	380
EUR_Zero_2Y / USD_Zero_2Y (covariance)	290
EUR_Zero_2Y / EUR/USD (covariance)	−35

(a) Calculate the correlation between EUR_Zero_2Y and USD_Zero_2Y.

(b) Calculate the correlation between EUR_Zero_2Y and EUR/USD. What does this mean economically?

(c) How does the VaR of a EUR bond change if the correlation between EUR and USD rates rises from $\rho = 0.7$ to $\rho = 0.9$? Give a qualitative answer with reasons.

(d) The calculator displays variances in units of $\times 10^{-8}$. What is the standard deviation of the log-return for EUR_Zero_2Y in percent?

Exercise 11 — *H — Risk Factor Mapping*

The calculator assigns each position to the *nearest* tenor in the factor list.

(a) A bond pays a coupon at $t = 1.5$ years. To which of the five EUR tenors (0.25Y, 1Y, 2Y, 5Y, 10Y) is it mapped?

(b) Show that this mapping makes the sensitivity matrix sparse: a bond with 4 cash flows may map to only 3 different factors — why?

(c) **Rho mapping as an alternative:** instead of the nearest tenor, one could interpolate linearly onto the two neighbouring tenors. For $t = 1.5$Y with sensitivity $\delta = 1000$ this would give:

$$\delta_{1\text{Y}} = 1000 \cdot \frac{2 - 1.5}{2 - 1} = 500, \quad \delta_{2\text{Y}} = 1000 \cdot \frac{1.5 - 1}{2 - 1} = 500.$$

What would be the advantage of this approach? Which condition must the interpolation satisfy so that the total present value is preserved?

(d) A bond with rating A has a cash flow at $t = 3$ years. The calculator maps it to CS_A_5Y (the nearest CS tenor is 5Y). Describe in which direction this error biases the VaR.

4 Block D: VaR Calculation and Sub-Additivity

Exercise 12 — *E — Basic VaR Calculation*

A portfolio consists of two positions with the following figures:

	Position A	**Position B**
Sensitivity δ	50,000	−30,000
Assigned factor	EUR_Zero_5Y	EUR_Zero_5Y
Variance σ^2 ($\times 10^{-8}$)	550	550

(a) Calculate the aggregated weight $w = \delta_A + \delta_B$.

(b) Calculate $\sigma_P^2 = w^2 \cdot \sigma^2$.

(c) Calculate $\text{VaR}_{99\%} = z_{0.99} \cdot \sigma_P$ with $z_{0.99} = 2.326$.

(d) Why is the VaR smaller than $|\text{VaR}_A| + |\text{VaR}_B|$, even though both positions map to the same risk factor? (*Keyword: natural hedging*)

Exercise 13 — *M — Two-Factor Portfolio*

A portfolio has sensitivities to two risk factors:

$$w_1 = 80{,}000 \text{ (EUR_Zero_5Y)}, \qquad w_2 = 60{,}000 \text{ (USD_Zero_5Y)}.$$

The relevant entries of the covariance matrix ($\times 10^{-8}$):

$$\Sigma_{11} = 550, \quad \Sigma_{22} = 480, \quad \Sigma_{12} = 350.$$

(a) Calculate $\sigma_P^2 = w_1^2\Sigma_{11} + 2w_1w_2\Sigma_{12} + w_2^2\Sigma_{22}$.

(b) Calculate $\mathrm{VaR}_P = z_{0.99} \cdot \sigma_P$.

(c) Calculate VaR_1 and VaR_2 from the diagonal entries.

(d) Calculate the diversification effect: $\Delta\mathrm{VaR} = (\mathrm{VaR}_1 + \mathrm{VaR}_2) - \mathrm{VaR}_P$.

(e) What would VaR_P be if $\rho_{12} = 0$? And if $\rho_{12} = 1$? Interpret the extreme cases.

Exercise 14 — *M — Proving Sub-Additivity*

Show algebraically for the case of *two* factors with variances σ_1^2, σ_2^2 and correlation $\rho \in [-1, 1]$:

$$\mathrm{VaR}_P = z_\alpha\sqrt{w_1^2\sigma_1^2 + 2\rho w_1w_2\sigma_1\sigma_2 + w_2^2\sigma_2^2} \;\le\; z_\alpha(|w_1|\sigma_1 + |w_2|\sigma_2) = \mathrm{VaR}_1 + \mathrm{VaR}_2.$$

(a) First show that it suffices to prove $A^2 \le (B + C)^2$ with $A = \sqrt{w_1^2\sigma_1^2 + 2\rho w_1w_2\sigma_1\sigma_2 + w_2^2\sigma_2^2}$, $B = |w_1|\sigma_1$, $C = |w_2|\sigma_2$.

(b) Show that $A^2 = B^2 + 2\rho\,\mathrm{sgn}(w_1w_2)\,BC + C^2$.

(c) Using $\rho \le 1$ and $\mathrm{sgn}(w_1w_2) \le 1$, conclude that $A^2 \le (B + C)^2$.

(d) When does equality hold? When is $\mathrm{VaR}_P = \mathrm{VaR}_1 + \mathrm{VaR}_2$?

Hint

For $X, Y \ge 0$ the inequality $\sqrt{X} \le Y$ is equivalent to $X \le Y^2$. Use $\rho \le 1$ in the decisive step.

Exercise 15 — *H — Diagonal-Block Methodology*

The calculator computes individual VaRs from the *same* diagonal blocks of the global matrix. Why is this crucial for sub-additivity?

(a) Suppose the individual VaR of the bonds is computed from a *separate*, tab-specific covariance matrix $\tilde{\boldsymbol{\Sigma}}_{\mathrm{B}}$, and the portfolio VaR from the global $\boldsymbol{\Sigma}$. Construct a counterexample in which $\mathrm{VaR}_P > \mathrm{VaR}_{\mathrm{Bonds}} + \mathrm{VaR}_{\mathrm{Swaps}}$. (*Hint: choose* $\tilde{\boldsymbol{\Sigma}}_B$ *such that* VaR_{Bonds} *is very small but the corresponding block in* $\boldsymbol{\Sigma}$ *is much larger.*)

(b) Explain in your own words why using the *same* matrix for all VaRs ensures sub-additivity.

(c) What is the regulatory significance of sub-additivity? Why is a risk measure that violates sub-additivity problematic for banks?

5 Block E: Calculator Experiments

Exercise 16 — *E — First Run*

Start the calculator with the default values and load `market_data_251days.csv`.

(a) Record the current EUR, USD, GBP and CHF zero rates for 5 years (from the last CSV row).

(b) Calculate all four product tabs (bonds, swaps, CCS, FX). For each tab, record: total PV, individual VaRs and total VaR (with correlation).

(c) Calculate the portfolio VaR in the *Portfolio VaR* tab. Is the portfolio VaR smaller than the sum of the tab VaRs? Calculate the diversification effect in percent.

(d) Switch to the *Matrices & Cash Flows* tab. How many positions does the total portfolio contain?

Exercise 17 — *M — Sensitivity Analysis*

Investigate the influence of various parameters on the VaR.

(a) Maturity: set all bonds to a maturity of 1 year. Calculate the bond VaR. Repeat with 10 years. Explain the difference using the formula $\ell \approx -t \cdot \Delta r$.

(b) Rating: change bond 1 from rating A to *no spread*. How does the individual VaR change? Why can it change even though the PV hardly changes?

(c) Currency: change swap 2 from USD to CHF. Does the diversification effect between the swaps change? Why?

(d) Offsetting position: add a second bond with the same parameters as bond 1 but with a negative sign (short sale: negative coupon). What do you expect for the total VaR? Check with the calculator.

Exercise 18 — *H — Matrix Analysis*

Export the global covariance matrix as a CSV file (*Matrices & Cash Flows* → *Export matrix as CSV*).

(a) Open the CSV in Excel or Python. Calculate the *correlation matrix* from the covariance matrix:

$$\rho_{ij} = \frac{\Sigma_{ij}}{\sqrt{\Sigma_{ii} \cdot \Sigma_{jj}}}.$$

(b) Identify the three pairs of risk factors with the *highest* positive correlation. Are these the pairs you expected in Exercise 9c?

(c) Are there pairs with negative correlation? Which ones, and how can they be explained economically?

(d) Manually calculate the VaR of a portfolio containing only one EUR bond (notional 100,000 EUR, rating A, 5Y) and compare it with the calculator result. (*Hint: also export the cash flows as CSV.*)

Hint

Python code for the correlation matrix:

```
import pandas as pd, numpy as np
cov = pd.read_csv("kovarianzmatrix_29x29.csv",
                  index_col=0, sep=";",
                  skiprows=1)
std = np.sqrt(np.diag(cov.values))
corr = cov.values / np.outer(std, std)
```

Formula Sheet

Quantity	Formula
Discount factor	$\mathrm{DF}(t) = e^{-(r(t)+s(t))\,t}$
Linear interpolation	$r(t) = r(t_k)\,\frac{t_{k+1}-t}{t_{k+1}-t_k} + r(t_{k+1})\,\frac{t-t_k}{t_{k+1}-t_k}$
Log-return DF	$\ell(d) = \ln\!\big(\mathrm{DF}_d/\mathrm{DF}_{d-1}\big) \approx -t\cdot\Delta r$
Approximation (small Δr)	$\ell \approx -t\cdot\big[\Delta r(t) + \Delta s(t)\big]$
Bond present value	$\mathrm{PV} = \frac{1}{S}\sum_j \mathrm{CF}_j\cdot\mathrm{DF}(t_j)$
Swap floating leg	$\mathrm{PV}_{\text{float}} = N\cdot(1-\mathrm{DF}(T))$
Sensitivity	$\delta_j = \partial\mathrm{PV}/\partial\mathrm{DF}(t_j) = \mathrm{CF}_j/S$
Covariance estimator	$\Sigma_{ij} = \frac{1}{T-2}\sum_{d=1}^{T-1}(\ell_i(d)-\bar\ell_i)(\ell_j(d)-\bar\ell_j)$
Portfolio variance	$\sigma_P^2 = \mathbf{w}^\top\boldsymbol{\Sigma}\,\mathbf{w}$
VaR (normal distribution)	$\mathrm{VaR}_\alpha = z_\alpha\cdot\sigma_P$
Quantiles	$z_{0.95} = 1.645, \quad z_{0.99} = 2.326$
Correlation	$\rho_{ij} = \Sigma_{ij}/\sqrt{\Sigma_{ii}\Sigma_{jj}}$
Two-factor variance	$\sigma_P^2 = w_1^2\sigma_1^2 + 2\rho w_1 w_2\sigma_1\sigma_2 + w_2^2\sigma_2^2$
Sub-additivity	$\mathrm{VaR}_P \le \sum_k \mathrm{VaR}_k$
Diversification effect	$\Delta\mathrm{VaR} = \sum_k \mathrm{VaR}_k - \mathrm{VaR}_P \ge 0$

Risk factors (selection)	Block
EUR/USD/GBP/CHF_Zero_3M/1Y/2Y/5Y/10Y	Indices 0–19
EUR/USD, EUR/GBP, EUR/CHF	Indices 20–22
CreditSpread_A_1Y/5Y/10Y	Indices 23–25
CreditSpread_BBB_1Y/5Y/10Y	Indices 26–28